RADIANT HEAT TRANSMISSION FROM GASES IN TUBES

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January 16, 1948

Professor J. S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Master of Science in Naval Construction and Engineering, we submit herewith a thesis entitled "Radiant Heat Transmission from Gases in Tubes".

Respectfully,

RADIANT HEAT TRANSMISSION FROM GASES IN TUBES

by

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Submitted in Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE IN NAVAL CONSTRUCTION AND ENGINEERING

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1948

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NOMENCLATURE

A	Area of surface in square feet.
$\mathcal{F}$	A dimensionless factor to allow for interchange between gas tube surfaces expressed in terms of ϵ's, A's, α's, sight factors.
L	Beam lengths for gas radiatipn, feet.
P	Partial pressure of gas in atmospheres, P_c for carbon dioxide, P_w for water vapor, P_t total pressure of gas
q	Rate of heat transfer by radiation; Btu per hour.
T	Absolute temperature; Rankine; Fahrenheit absolute (460 + degrees Fahrenheit); T_s of surface; T_F of gas; T_1 gas in; T_2 gas out; T_{si} surface at gas inlet; T_{so} surface at gas outlet.
t	Temperature, degrees Fahrenheit; for subscripts see T.

GREEK

α	Absorptivity, dimensionless.
ϵ	Emissivity, dimensionless.
σ	Dimensionless constant in Stefan-Boltzmann law.

SUMMARY

A method is presented for obtaining an estimate of the average radiant heat transfer rate from gaseous carbon dioxide and water vapor in steady flow through long thin tubes which is brief and of sufficient accuracy for engineering use.

Recommendations are based on calculations of the true average heat transfer rate under the following conditions:

(a) there is no axial heat exchange,

(b) there is complete normal mixing of the gases,

(c) radiation is the only mechanism of heat transmission present.

The true average heat transfer rate was calculated from the basic data on emissivities of carbon dioxide and water vapor according to Egbert and Hottel (3). Three methods of calculating the average heat transfer rate were selected and compared with the true average for a wide range of conditions. From these the method best suited for adaptation to engineering use was chosen. Since correlation of all the variables entering was impossible a systematic elimination of these variables was effected in such a manner as to maintain the error so introduced to a minimum.

The final result of the calculation is the recommendation for use of a geometric mean method for calculating the average radiant heat transfer rate in one step together with a correction to be applied to the results of this method. A correction curve is given in figure I-A

for a gas temperature change of 1000°F. Figure I-B gives a multiplier to be used for gas temperature changes other than 1000°F.

The geometric mean method is defined from the equation

$$(q/A)\text{ave} = \sigma \mathcal{F}\,\text{ave}\left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]\text{ave}$$

$\mathcal{F}$ ave is calculated at the arithmetic mean gas and surface temperatures.

$\left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]$ ave is taken as the geometric mean of the inlet and outlet values of this quantity.

The range of possible error in calculating the average heat transfer by this method is presented for four arithmetic mean gas temperatures in figure II. The symmetry of these bands about zero shows the recommended correction curve to be a good average for all possible conditions. The amount of error possible in the use of the recommended method is low in the range of high temperature industrial applications. It is excessive, however, for arithmetic mean gas temperatures as low as 1500°F together with gas temperature changes of 1500°F. When the difference between gas and surface temperatures is less than 500°F the method is no longer applicable.

The limiting conditions on which recommendations have been based, of a long thin tube and of no heat transmission by convection, do not exist perfectly in any practical application. They do appear in many applications to a near approximation. The extent to which the approximation may be made depends on the particular situation and the designers judgement.

FIGURE I-A
RECOMMENDED CORRECTION FOR AVERAGE VALUES OF
q/A FROM GEOMETRIC MEAN METHOD FOR $t_1 - t_2 = 1000°$ F

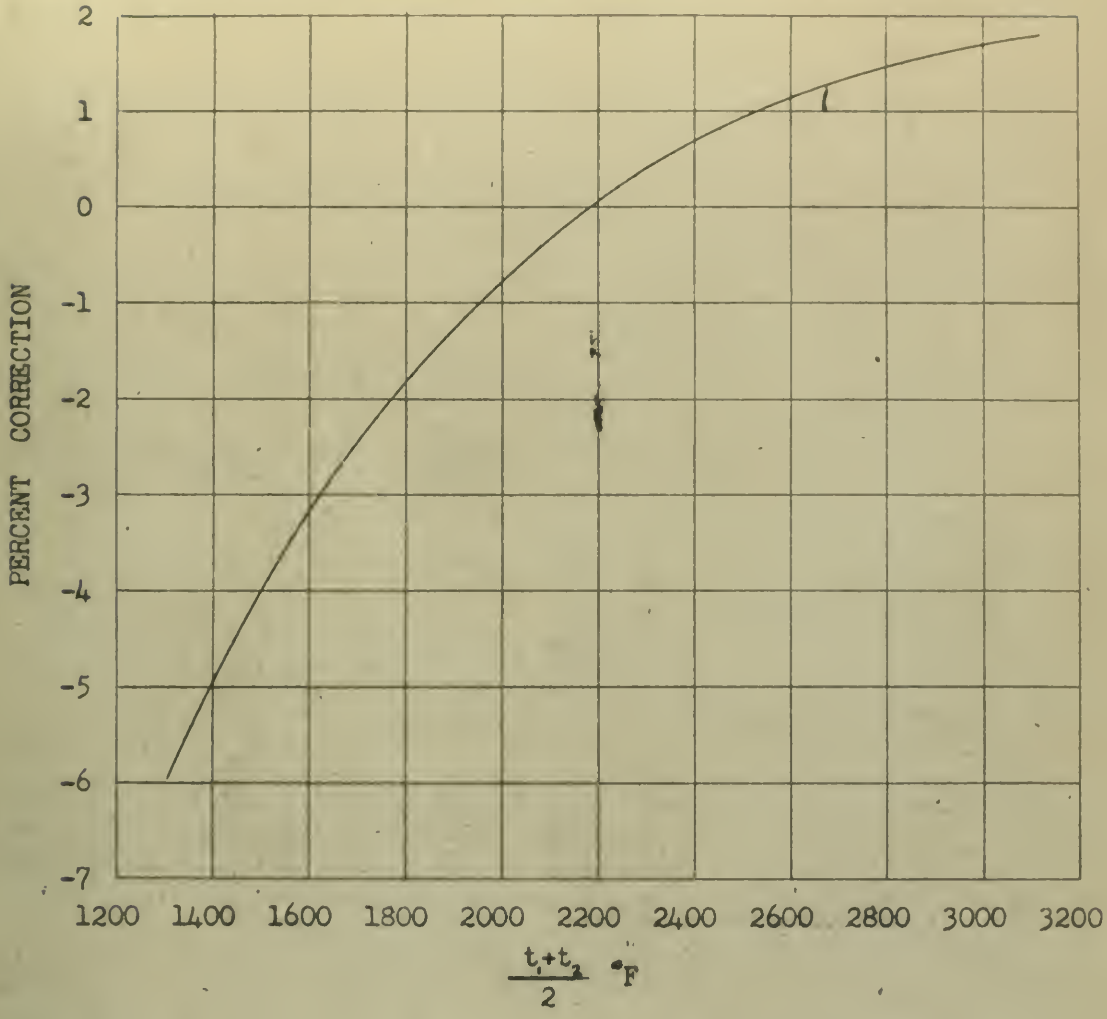
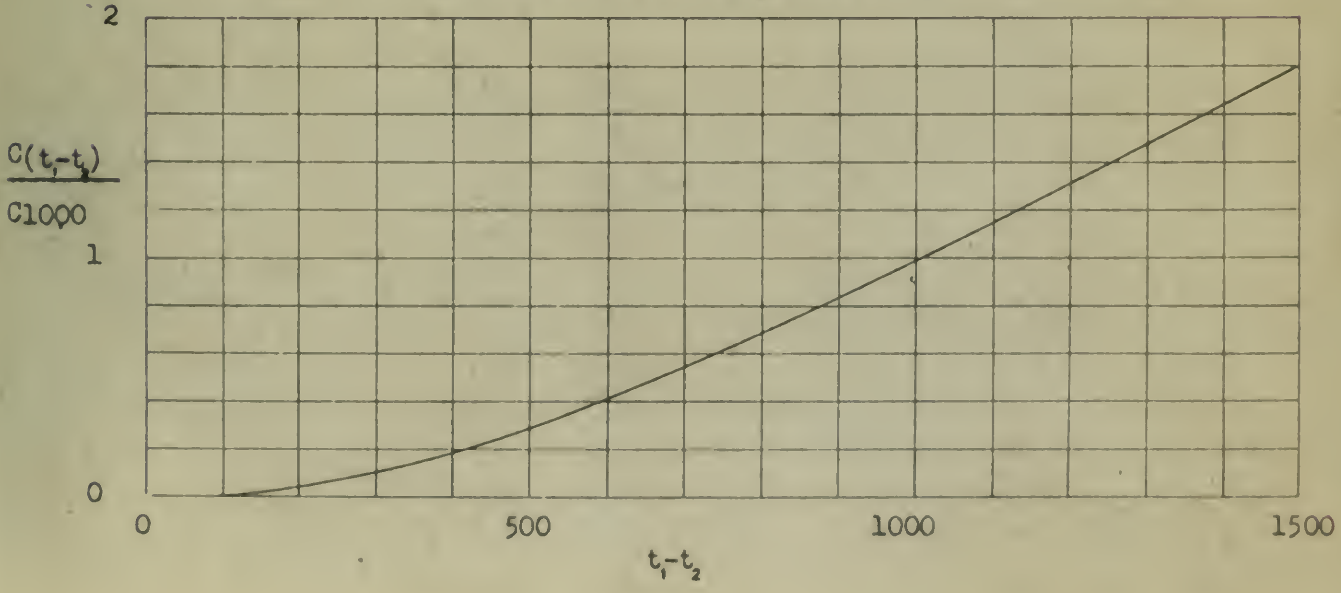


FIGURE I-B
RATIO OF RECOMMENDED CORRECTION AT ANY $t_1 - t_2$
TO CORRECTION AT $t_1 - t_2 = 1000°$ F



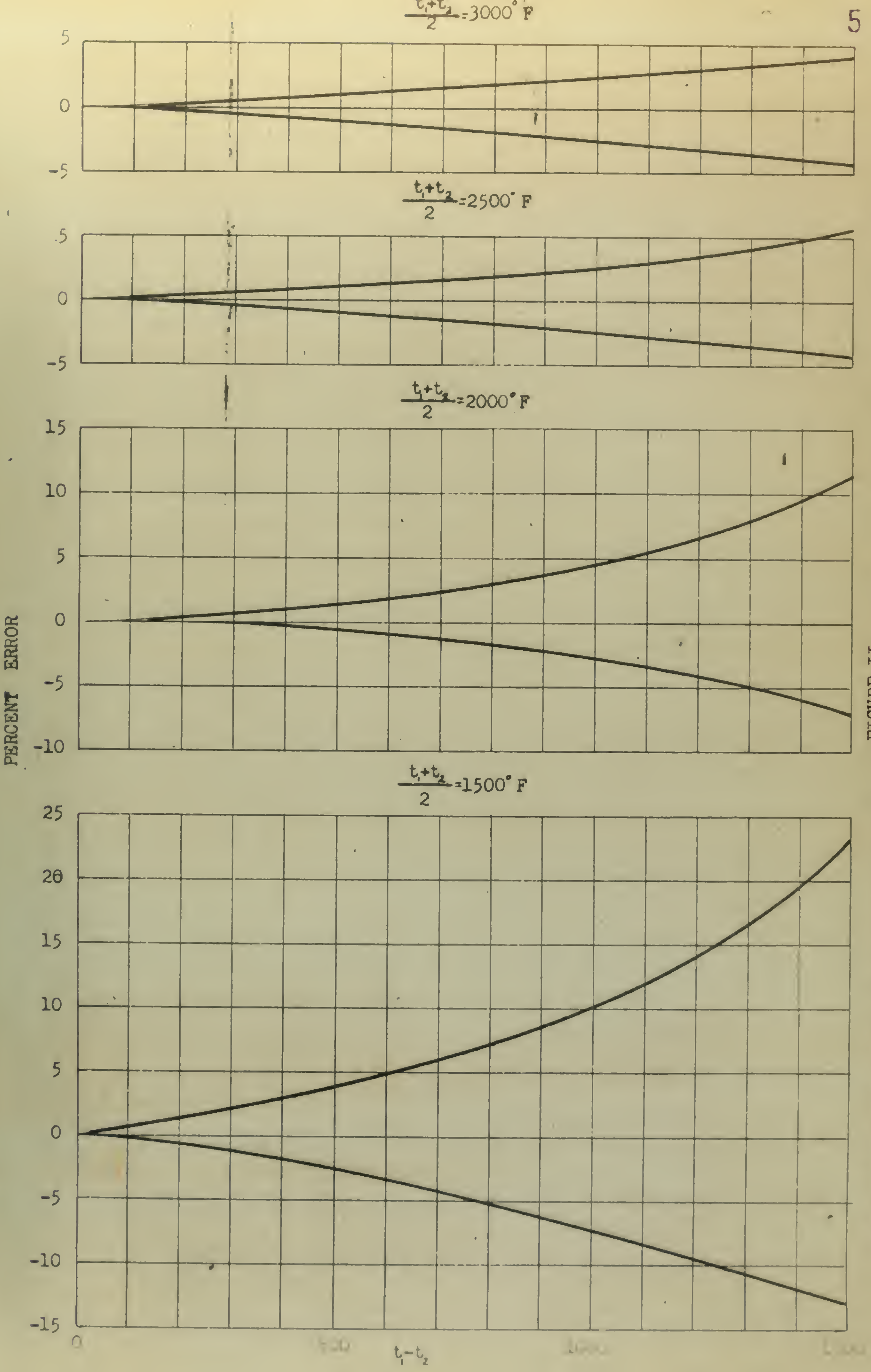


FIGURE II
BANDS OF POSSIBLE ERROR IN AVERAGE VALUE OF q/A FROM GEOMETRIC MEAN METHOD AND RECOMMENDED CORRECTION

INTRODUCTION

The calculation of average radiant heat transfer from hot gases flowing through a tube is a rather long, tedious process, and one that is frequently encountered in the design of heat transfer equipment. Therefore, any simplification of this problem that does not sacrifice the accuracy of the results would be desirable.

The radiant heat transfer relation at a point is expressed by a fourth power equation $q/A\sigma = \mathcal{F}(T_F^4 - T_s^4)$. Data for the solution of this equation is available in a paper by Egbert and Hottel (3) in which the calculation of gas emissivity and absorptivity is discussed. To evaluate the average radiant heat transfer this equation must be solved on a point to point basis, calculating the emissivity and absorptivity of the gas at a number of points along the range desired, and integrating to get the average radiant heat transfer.

A theoretical approach to a reduction of the work involved in this calculation was made by Hottel (1) in which he offered a logarithmic mean of gas and surface temperatures for the evaluation of an average gas temperature which will permit calculation of the average radiant heat transfer in one step. Eberhardt and Hottel (2) took the case of a billet reheat furnace and combined empirical data and theoretical investigation to develop a simpler relationship for the calculation of radiant heat transfer. A pseudo emissivity was defined and a geometric mean of the terminal fourth power differences combined with it to form this simplified relationship for the calculation of the average heat transfer.

This thesis is concerned with the theoretical approach to the solution of average radiant heat transmission from gases in a tube. A point to point calculation has been made along a long thin tube, and the true average radiant heat transfer obtained by an integration of these points. Three types of means are used to evaluate the fourth power temperature differences. The average radiant heat transfer calculated in this manner is compared to the true value and the best mean chosen. The necessary correction curves have been made for this type of mean and a curve of error drawn to demonstrate the limits of this method.

PROCEDURE

By direct calculation based on the emissivities of water vapor and carbon dioxide as presented by Egbert and Hottel (3) there is obtained the average heat transfer rate from gases in steady flow through a tube to the tube wall. Two general conditions are imposed to define the problem:

a. The tube shall have a length to diameter ratio of infinity which permits the assumptions of no axial heat exchange and complete mixing of the gases normal to the axis of flow.

b. The radiant heat transfer to tube walls shall be controlling, neglecting convection effects from gases to tube and conduction effects along the tube.

Following this, the average heat transfer rate is obtained by three different methods which are approximations only. Values obtained in this manner are compared with the true average to determine if any one of the methods is usable in engineering calculations.

Radiant heat exchange from a body of hot gas to the cold heat receiving surface is calculated from the fourth power temperature difference law and from consideration of the gas composition, size and configuration of the gas body and surface conditions. It may be expressed by the equation

$$(Q/A)=\left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]$$

in which $\mathcal{F}$, as developed by Hottel (4), allows for all of these considerations. To simplify the calculation of $\mathcal{F}$ certain specific condi-

ditions were imposed:

a. $\epsilon_s \quad 1$

b. The entire surface of the tube is heat receiving.

c. The emissivity of water vapor is calculated without correction for its partial pressure.

The term $\mathfrak{F}$ now becomes

$$\mathfrak{F} = \frac{\epsilon_G - \alpha_{Gs}\left(\frac{T_s}{T_F}\right)^4}{1 - \left(\frac{T_s}{T_F}\right)^4}$$

and
$$(q/A) = \left[\frac{\epsilon_G - \alpha_{Gs}\left(\frac{T_s}{T_F}\right)^4}{1 - \left(\frac{T_s}{T_F}\right)^4}\right] \left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right] \sigma$$

$$= \left[\epsilon_G \left(\frac{T_F}{100}\right)^4 - \alpha_{Gs}\left(\frac{T_s}{100}\right)^4\right]$$

The average heat transfer rate over a span of gas temperature from T_1 to T_2 is found from

$$(q/A)\text{ave} = \frac{T_1 - T_2}{\int_{T_2}^{T_1} \frac{dT}{q/A}}$$

In order to obtain this average heat transfer rate the heat transfer is calculated at several points between T_1 and T_2 and its reciprocal is plotted against gas temperature t_F. The integral is evaluated by the trapezoidal rule for integrating areas.

Such a method of estimating as is presented here requires that the basis on which the recommendations are made be as all inclusive as is

practical. Accordingly the following values of variables were chosen and the average heat transfer was calculated for all possible combinations of these, the only limitations being the range of the charts of gas emissivities, and the requirement that gas temperature exceed surface temperature.

Variables:

$$\Sigma PL = 2.0,\ 0.20,\ 0.06$$

$$\frac{P_w}{P_c + P_w} = 0,\ 0.5,\ 1$$

$$t_s = 100^\circ F,\ 1500^\circ F,\ 0.8\ t_F - 400$$

$$t_1 - t_2 = 500,\ 100,\ 1500$$

$$\frac{t_1 + t_2}{2} = 800^\circ F \text{ to } 3300^\circ F$$

The last variable, the arithmetic mean gas temperature, was used as the abcissa for these plots.

The three methods of computing an average value of the heat transfer rate are:

a. Geometric mean method

$$(q/A\sigma)_{ave} = \mathcal{F}_{ave}\left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]_{ave}$$

$\mathcal{F}_{ave}$ is calculated at the arithmetic mean gas and surface temperatures.

$\left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]_{ave}$ is the geometric mean of inlet and outlet values,

$$= \sqrt{\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_{so}}{100}\right)^4\right]\left[\left(\frac{T_2}{100}\right)^4 - \left(\frac{T_{si}}{100}\right)^4\right]}$$

b. Arithmetic mean method

$$(q/A\sigma)_{ave} \quad \mathcal{F}_{ave}\left[\left(\frac{T_{F,ave}}{100}\right)^4 - \left(\frac{T_{s,ave}}{100}\right)^4\right]$$

$\mathcal{F}$ ave is calculated at the arithmetic mean gas and surface temperatures

$T_{F},$ ave = arithmetic mean gas temperature

$T_{s},$ ave = arithmetic mean surface temperature

c. Log mean method

$$(q/A\sigma)\text{ave} = \mathcal{F}\ \text{ave}\left[\frac{T_{F},\text{ave}\ ^{4}}{100} - \frac{T_{s},\text{ave}\ ^{4}}{100}\right]$$

$\mathcal{F}$ave is calculated at the arithmetic mean surface temperature, $t_{s},$ ave and at $t_{F},$ ave = $t_{s},$ ave+log mean temperature difference between gas and surface

$$T_{F},\text{ave} = t_{F},\text{ave} + 460$$

$$T_{s},\text{ave} = t_{s},\text{ave} + 460$$

A comparison of these means was accomplished by evaluating the correction necessary to make the average heat transfer rate from the method equal to the true rate. These corrections were plotted against the arithmetic mean gas temperature for each method, all surface temperature conditions, $t_1 - t_2 = 1000^{o}F$, ΣPL 2.0 and each value of the water fraction, $\frac{P_w}{P_c + P_w}$, (figures III, IV, V).

Curves for each surface temperature condition were independent of each other and a correlation was impossible. The curves for different surface temperatures fell most closely together at each value of the water fraction for the geometric mean method. Accordingly this method was chosen for further development.

Curves of correction versus arithmetic mean gas temperature were plotted for all combinations of the variables for the geometric mean method, (figures VI-XIV). Average curves were drawn to represent all

surface temperature conditions and at each combination of ΣPL and water fraction were forced to cross the abcissa at the same arithmetic mean gas temperature for all three values of $t_1 - t_2$.

The ratios of the ordinates of these three average curves were averaged to make the ratio independent of gas temperature. The average curve for $t_1 - t_2 = 1000$ was held fixed and the other two average curves corrected for the average ratio. In turn the ratios so obtained were again averaged to make them independent of water fraction, (figure XVII).

The average correction curves for $t_1 - t_2 = 1000$ were plotted together for each ΣPL, (figure XV), and since no correlation was possible an average was drawn through each group, now independent of surface temperature and water fraction.

Finally these curves of average correction ratio versus $(t_1 - t_2)$, (figure XVII), and of average correction for $t_1 - t_2 = 1000$, (figure XVI), were examined from consideration of their usefulness and accuracy. Again it was decided that the slight loss of accuracy incurred by averaging the results to this point was necessary to avoid interpolation difficulties in the use of the curves. Average curves were again faired through each set to produce one recommended correction curve for $t_1 - t_2 = 1000$ and one recommended curve of ratio of correction at any $(t_1 - t_2)$ to the correction at $t_1 - t_2 = 1000$, (figures I-A, I-B).

In eliminating the variables ΣPL, water fraction, and surface temperature by such averaging as has been done to this point considerable error may be introduced into the average heat transfer rate. This has been studied and the actual error plotted for all combinations of the variables and for arithmetic mean gas temperatures of 1500, 2000, 2500,

and 3000 °F, (figures XVIII-XXI). To make such a plot useful the bands of possible error are lifted and presented with the recommended correction curves, (figure II).

RESULTS

The results of the calculations are presented in figures I-A, I-B, and II on pages 3 and 4 and in figures III through XXI on the following pages.

Figures I-A, I-B, and II are the final recommended correction curves to be used in calculating an average heat transfer rate by the geometric mean method and the bands of error possible in such a calculation. Figures III through XXI are the correction curves and error plots which lead to the recommendations.

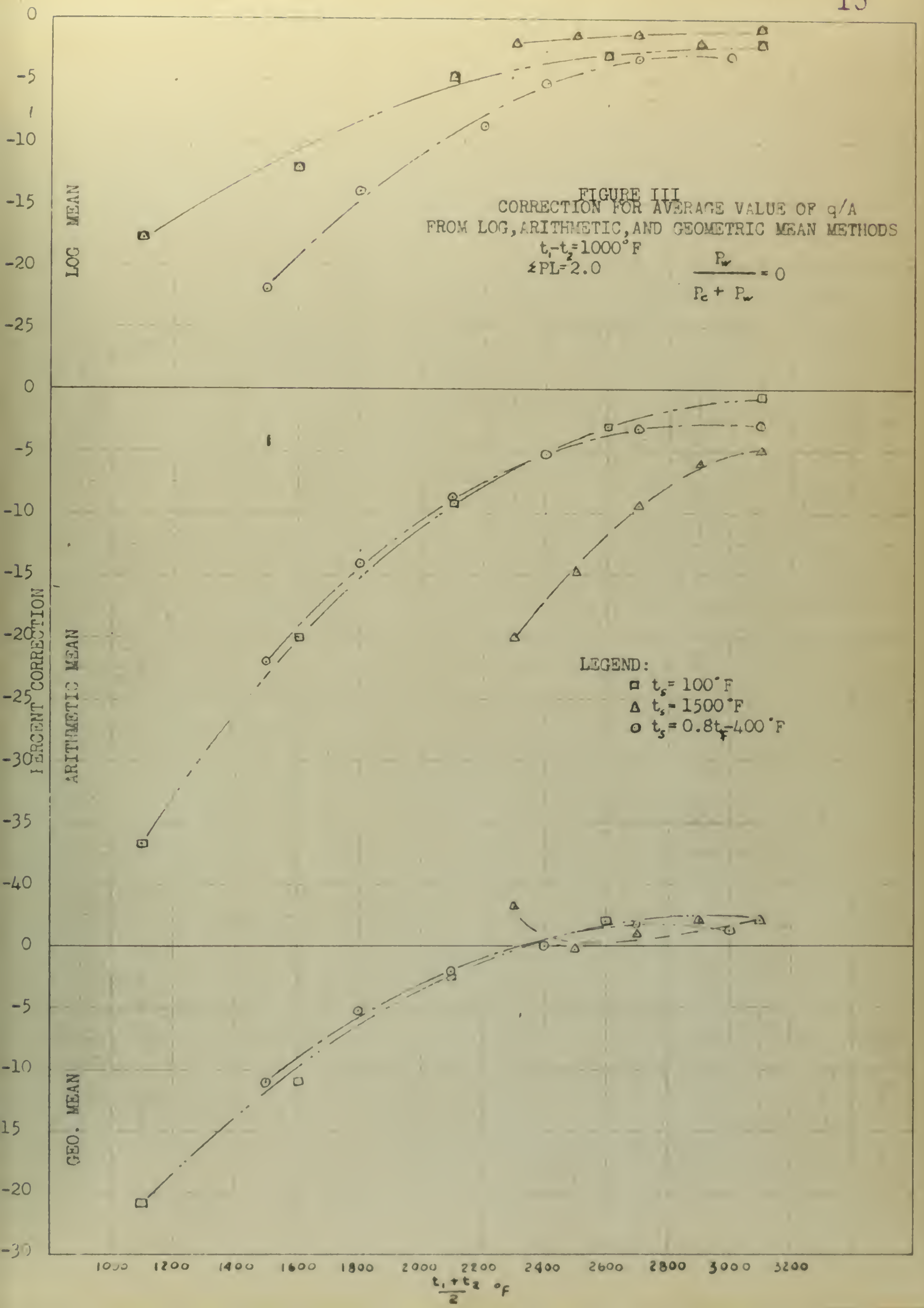


FIGURE III
CORRECTION FOR AVERAGE VALUE OF q/A
FROM LOG, ARITHMETIC, AND GEOMETRIC MEAN METHODS
$t_1 - t_2 = 1000°F$
$\Sigma PL = 2.0$
$$\frac{P_w}{P_c + P_w} = 0$$

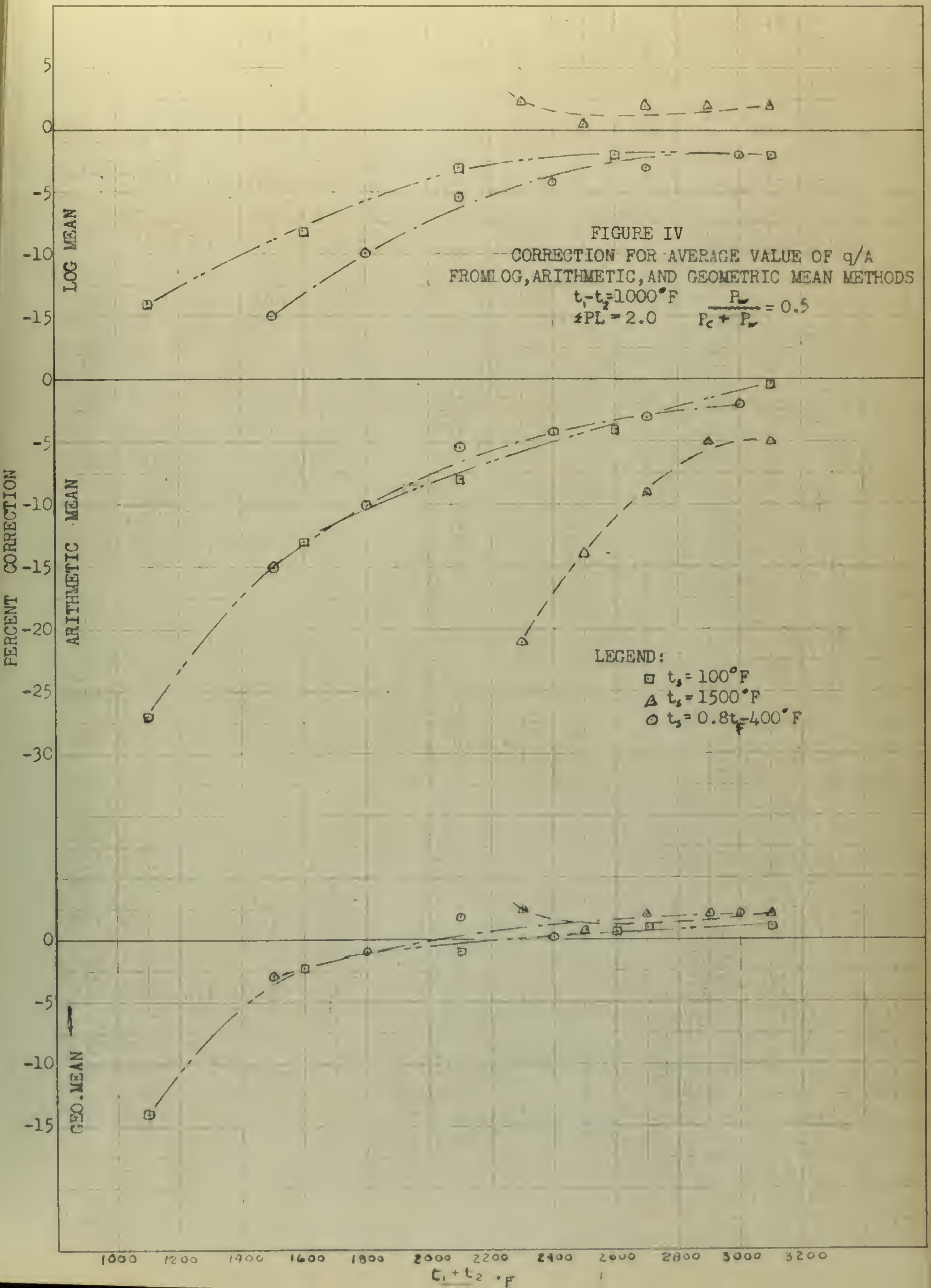


FIGURE IV

CORRECTION FOR AVERAGE VALUE OF q/A FROM LOG, ARITHMETIC, AND GEOMETRIC MEAN METHODS

$t_1 - t_2 = 1000°F$ $\frac{P_w}{P_c + P_w} = 0.5$

$\Sigma PL = 2.0$

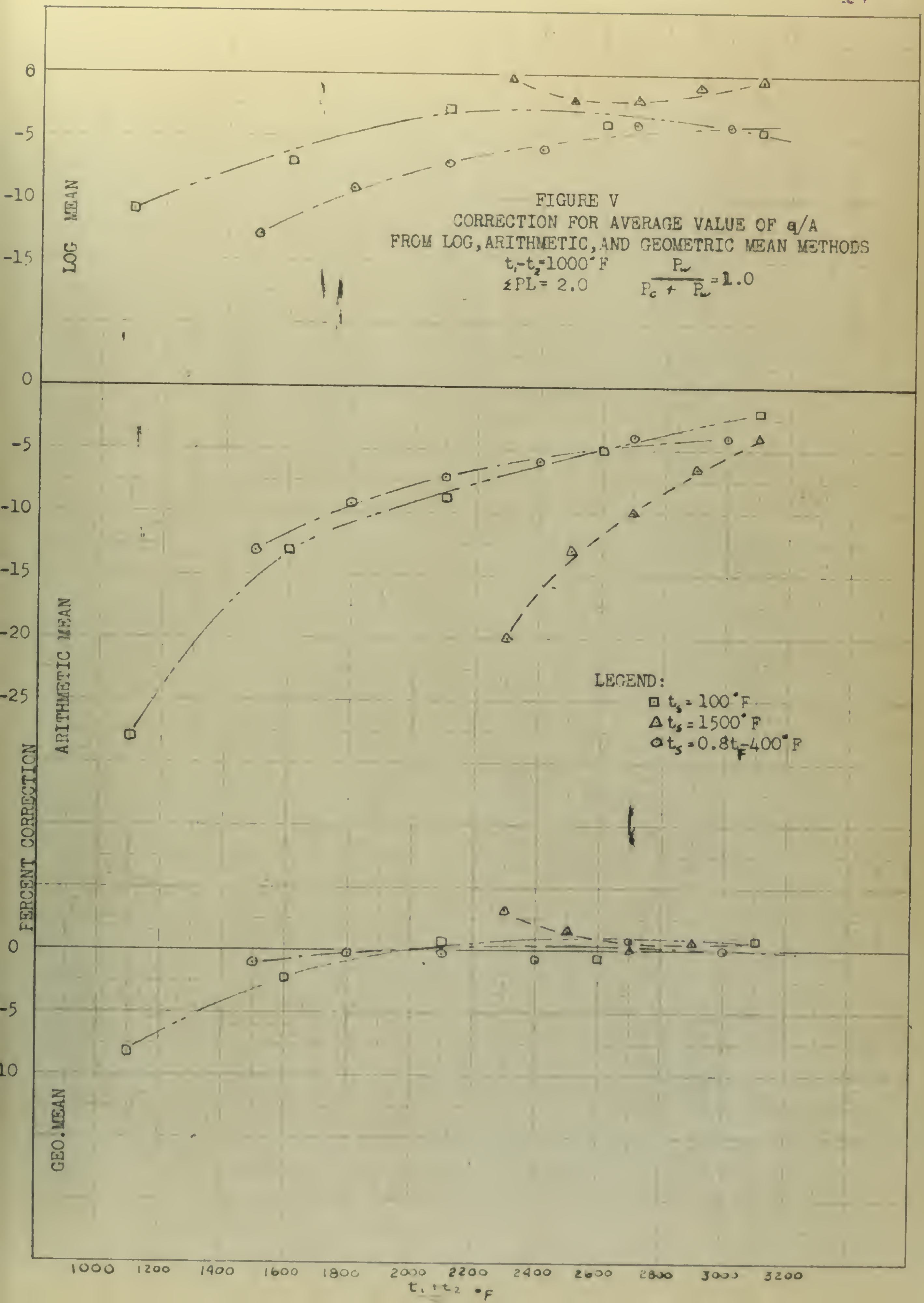


FIGURE V
CORRECTION FOR AVERAGE VALUE OF q/A
FROM LOG, ARITHMETIC, AND GEOMETRIC MEAN METHODS
$t_1 - t_2 = 1000°F$ $\frac{P_w}{P_c + P_w} = 1.0$
$\Sigma PL = 2.0$

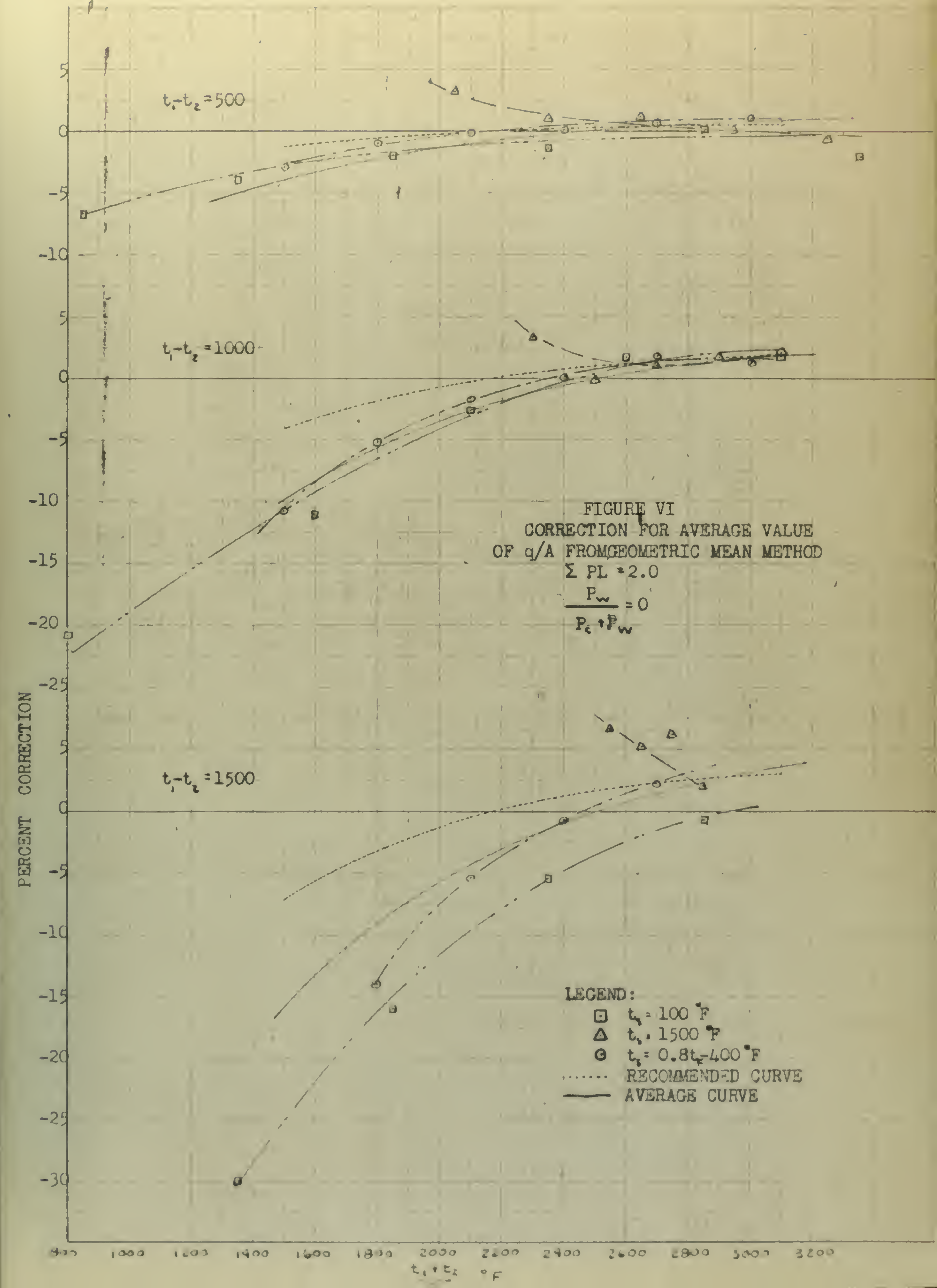


FIGURE VI
CORRECTION FOR AVERAGE VALUE
OF q/A FROMGEOMETRIC MEAN METHOD
$\Sigma PL = 2.0$
$\frac{P_w}{P_c + P_w} = 0$

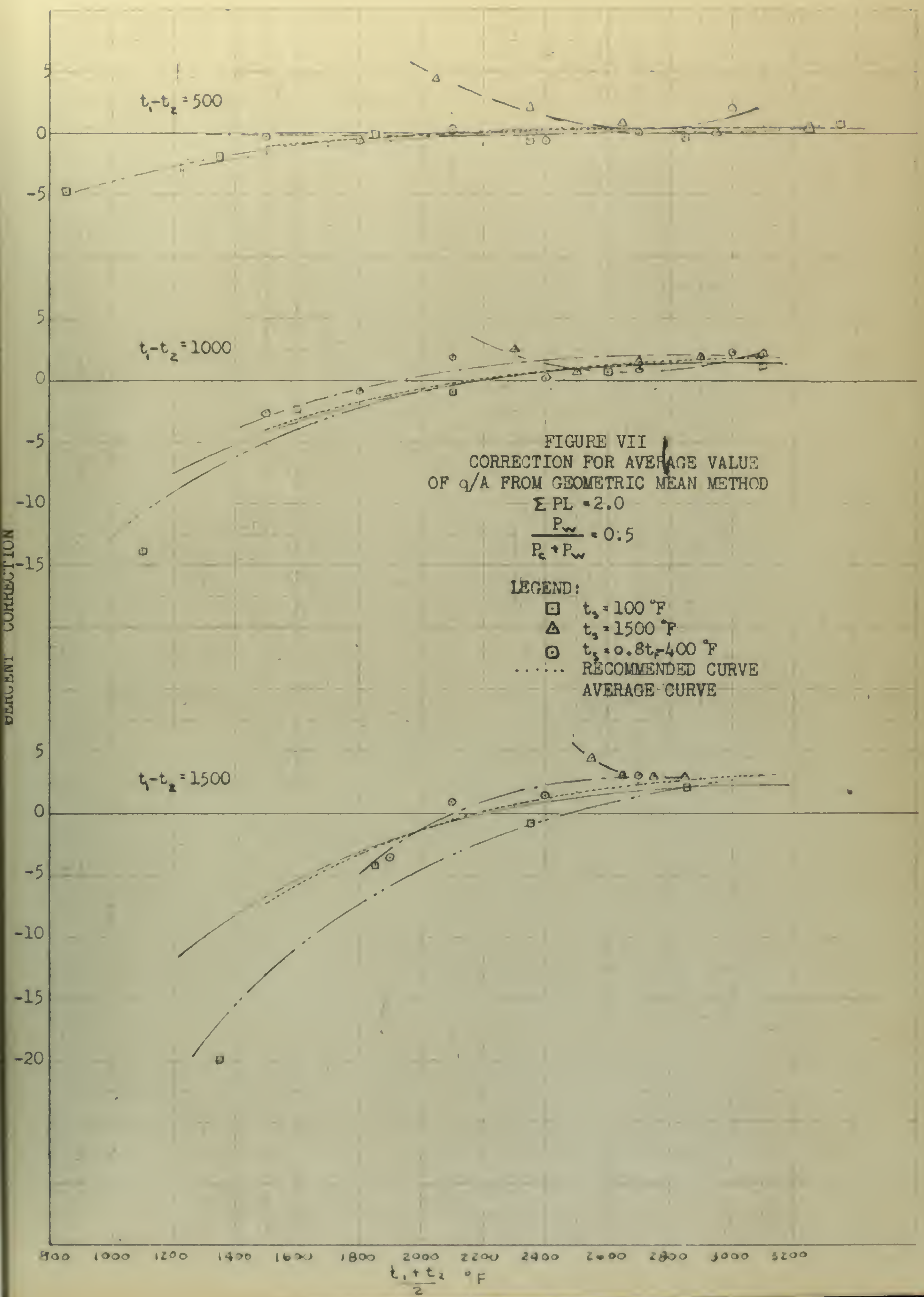


FIGURE VII
CORRECTION FOR AVERAGE VALUE
OF q/A FROM GEOMETRIC MEAN METHOD
$\Sigma PL = 2.0$
$\frac{P_w}{P_c + P_w} = 0.5$

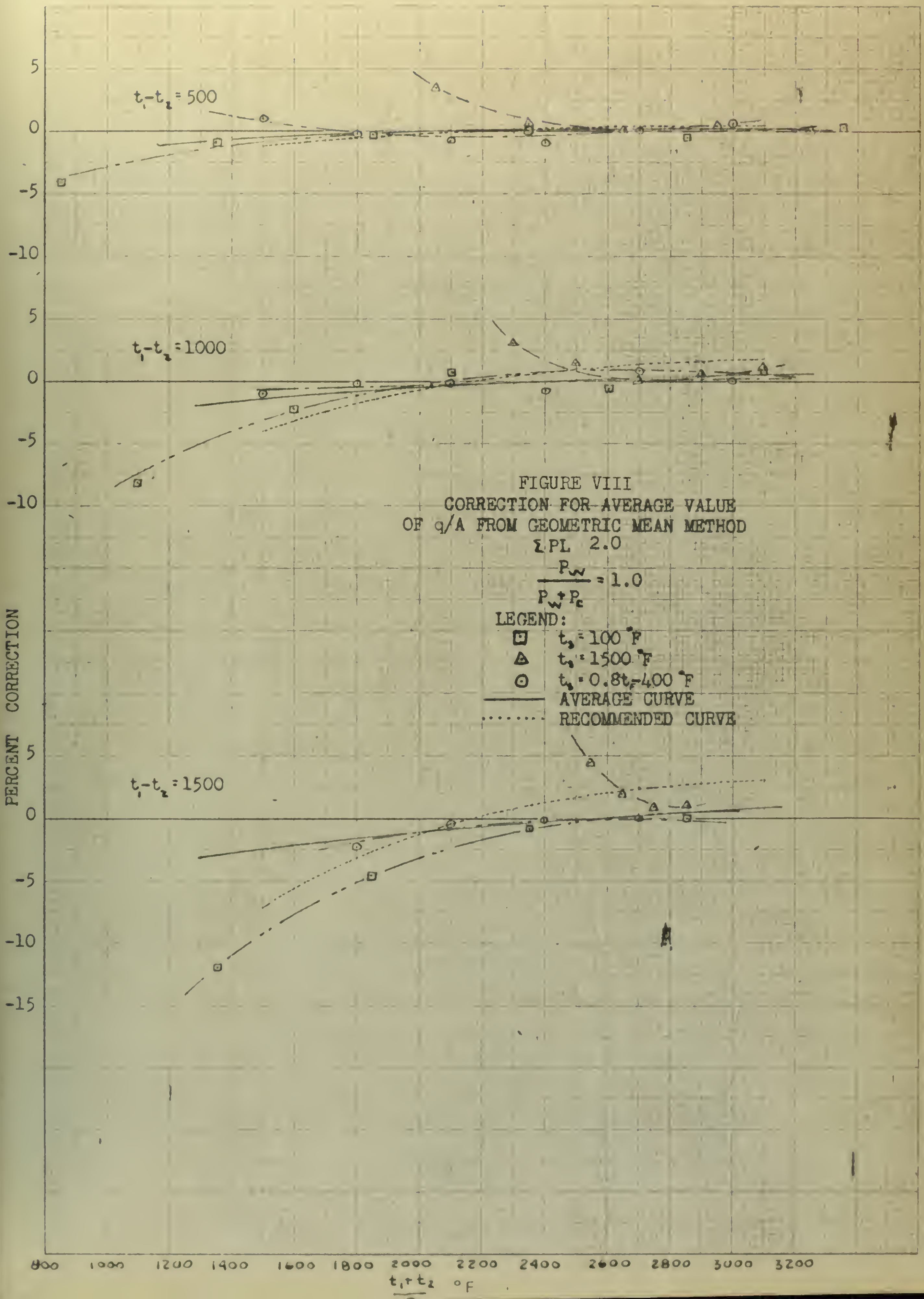


FIGURE VIII
CORRECTION FOR AVERAGE VALUE
OF q/A FROM GEOMETRIC MEAN METHOD
ΣPL 2.0

$$\frac{P_w}{P_w+P_c}=1.0$$

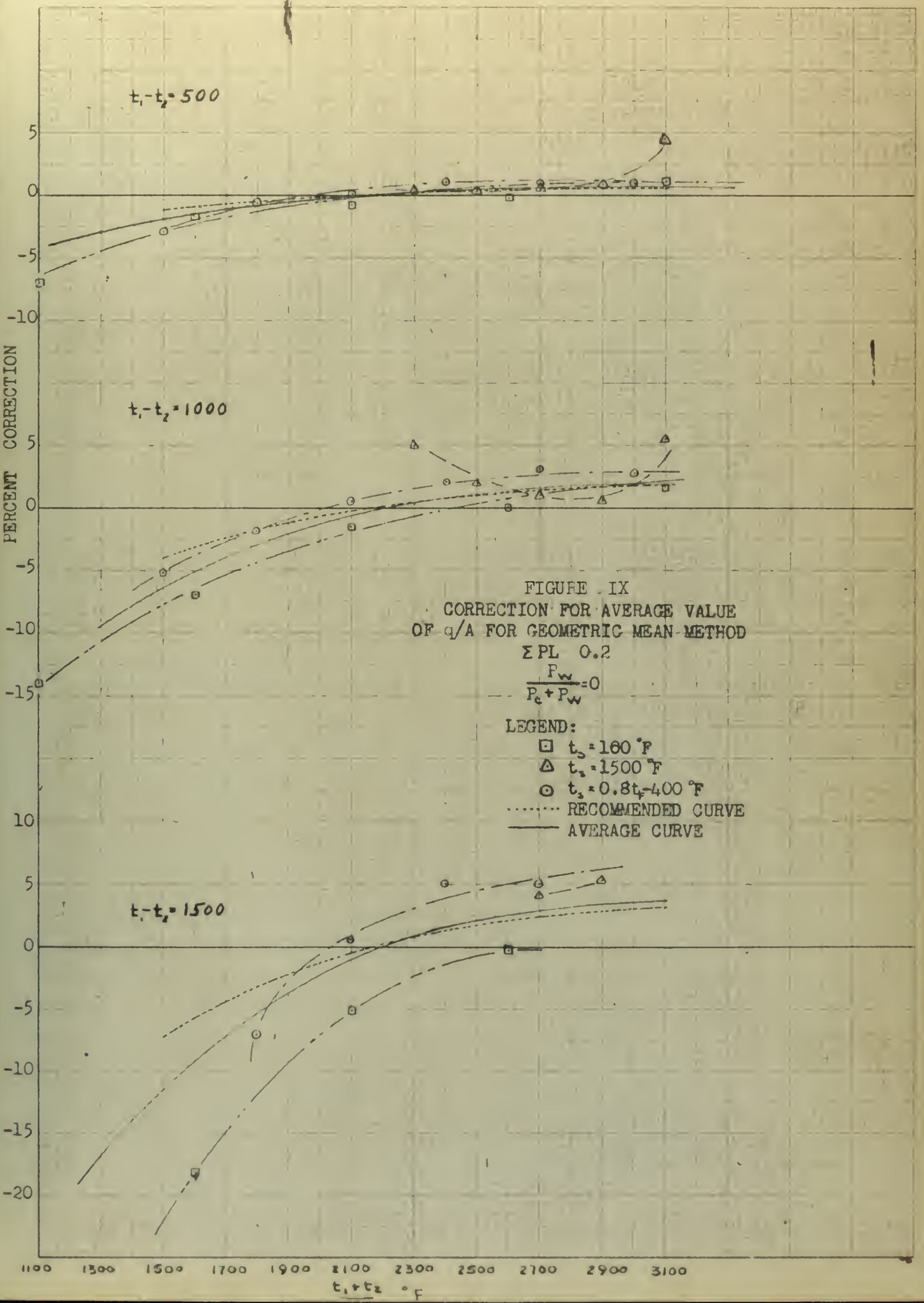


FIGURE IX
CORRECTION FOR AVERAGE VALUE
OF q/A FOR GEOMETRIC MEAN METHOD
$\Sigma PL \; 0.2$
$\frac{P_w}{P_c + P_w} = 0$

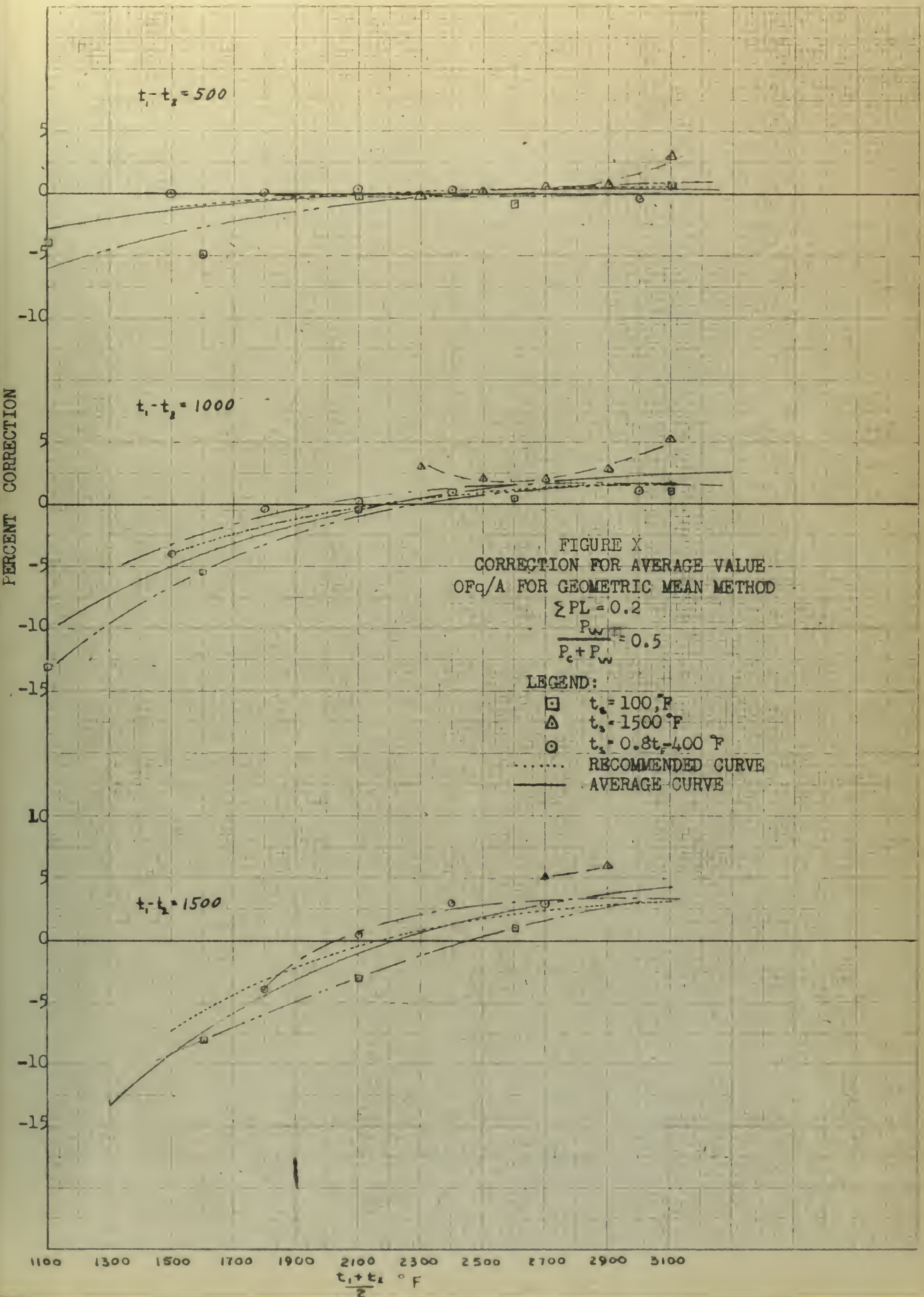


FIGURE X
CORRECTION FOR AVERAGE VALUE
OFq/A FOR GEOMETRIC MEAN METHOD
$\sum PL = 0.2$
$\frac{P_w}{P_c + P_w} = 0.5$

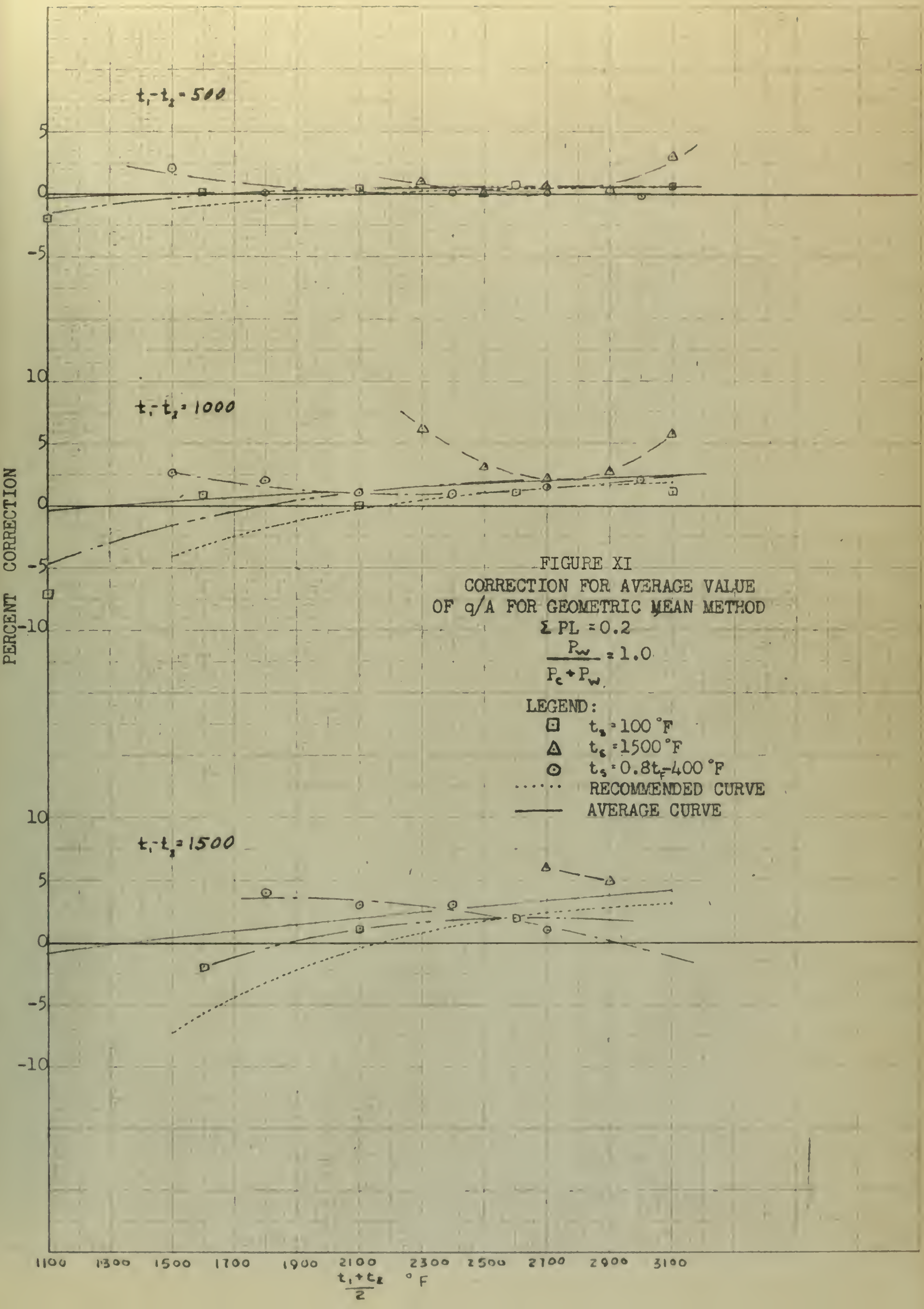


FIGURE XI
CORRECTION FOR AVERAGE VALUE
OF q/A FOR GEOMETRIC MEAN METHOD
$\Sigma PL = 0.2$
$\frac{P_w}{P_c + P_w} = 1.0$

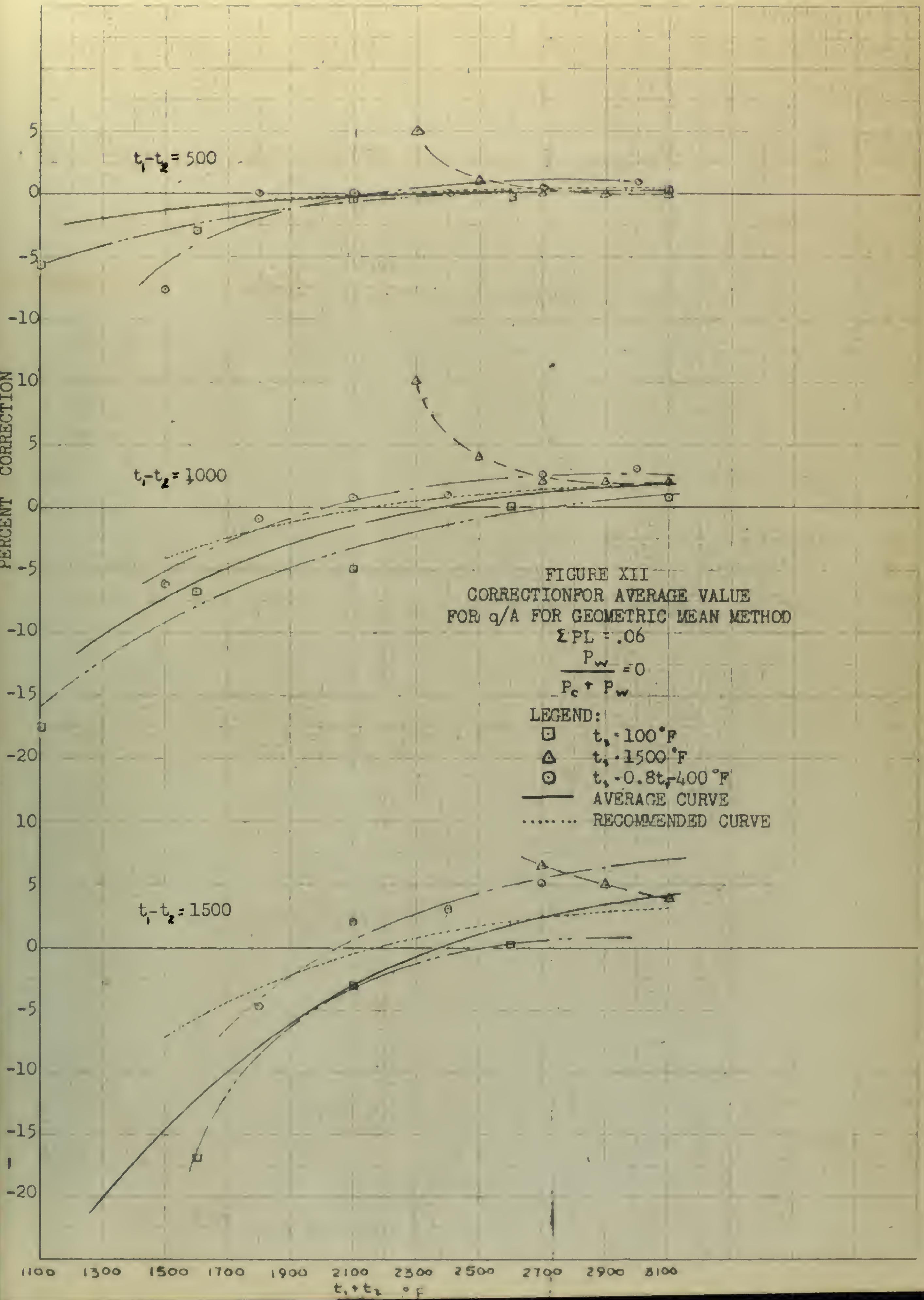


FIGURE XII
CORRECTIONFOR AVERAGE VALUE
FOR q/A FOR GEOMETRIC MEAN METHOD
$\Sigma PL = .06$
$\frac{P_w}{P_c + P_w} = 0$

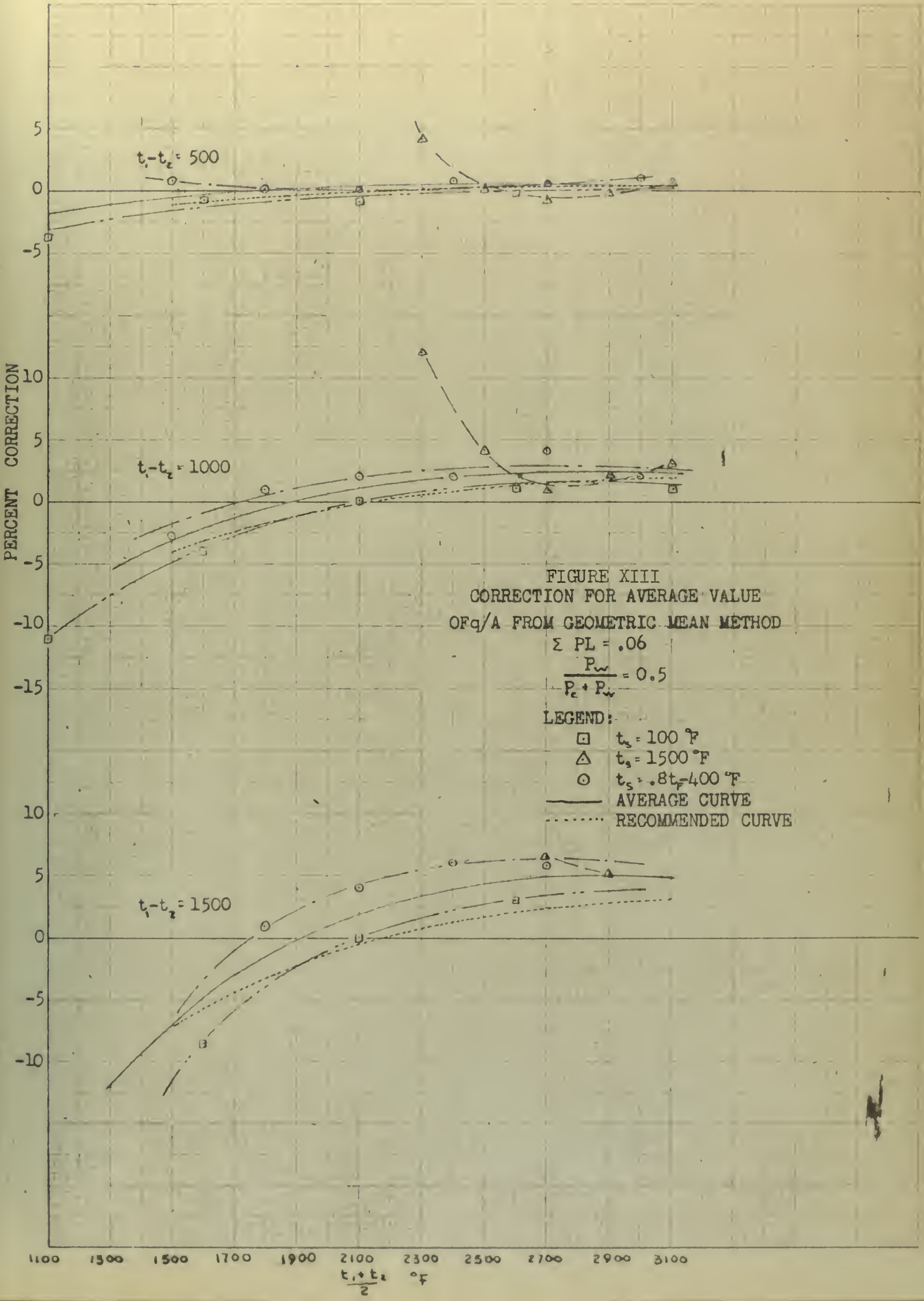


FIGURE XIII
CORRECTION FOR AVERAGE VALUE
OF q/A FROM GEOMETRIC MEAN METHOD
$\Sigma PL = .06$
$\frac{P_w}{P_c + P_w} = 0.5$

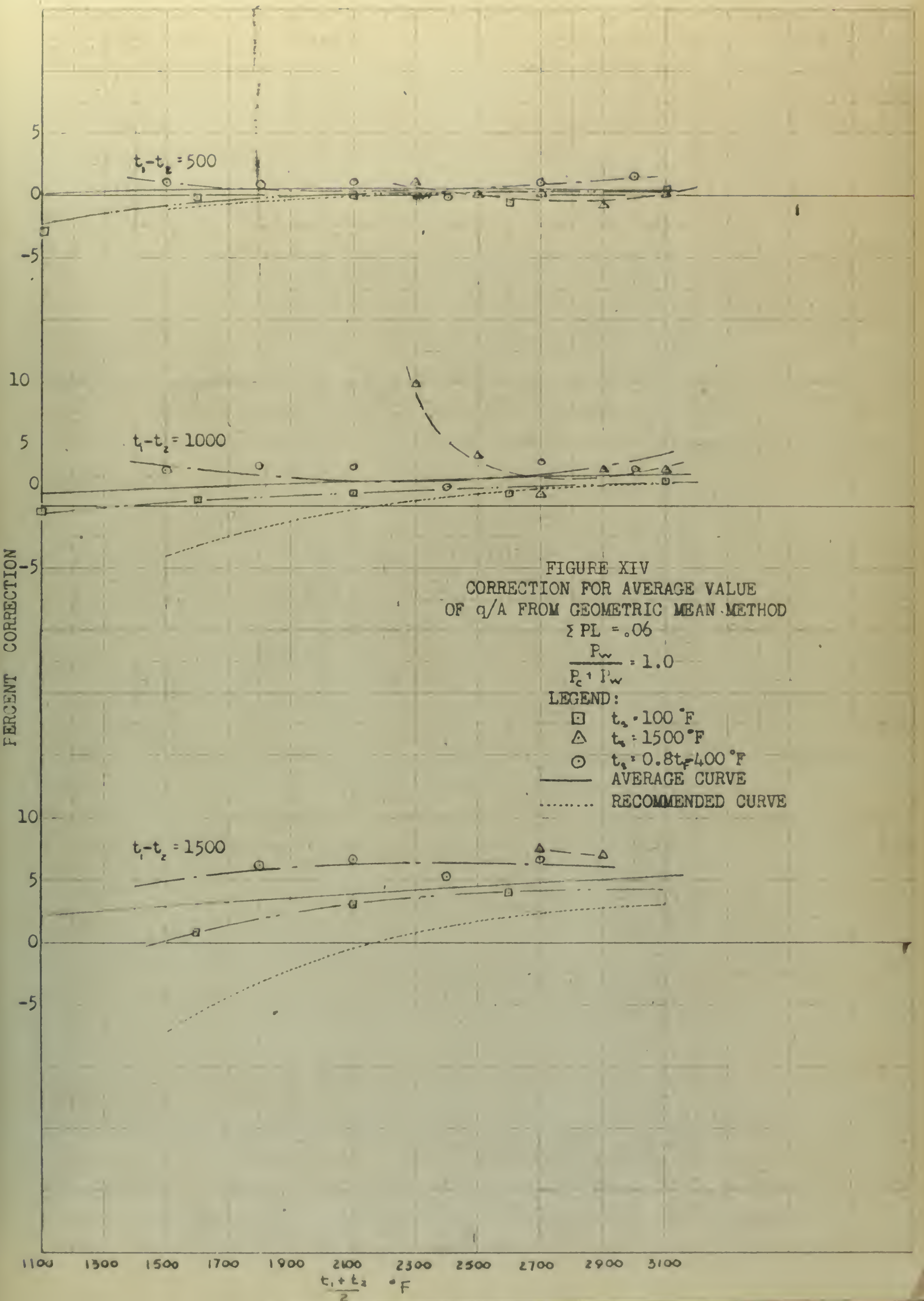


FIGURE XIV
CORRECTION FOR AVERAGE VALUE
OF q/A FROM GEOMETRIC MEAN METHOD
$\Sigma PL = .06$
$\frac{P_w}{P_c + P_w} = 1.0$

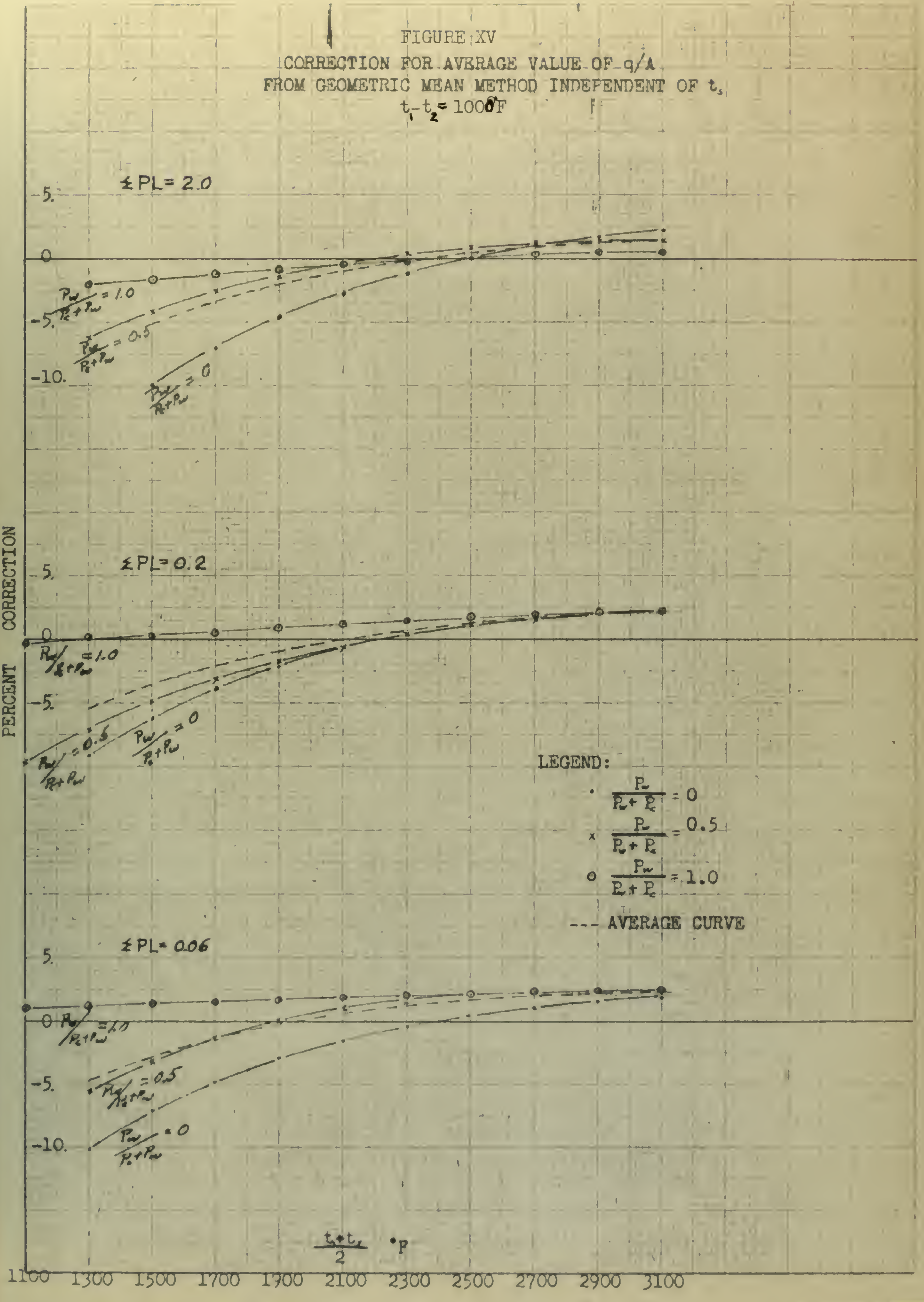


FIGURE XV
CORRECTION FOR AVERAGE VALUE OF q/A
FROM GEOMETRIC MEAN METHOD INDEPENDENT OF t_s
$t_1 - t_2 = 1000°F$

FIGURE XVI
CORRECTION TO AVERAGE VALUE OF q/A FROM GEOMETRIC MEAN METHOD INDEPENDENT OF t_s AND $\frac{P_w}{P_w + P_c}$
$t_1 - t_2 = 1000^{0}F$

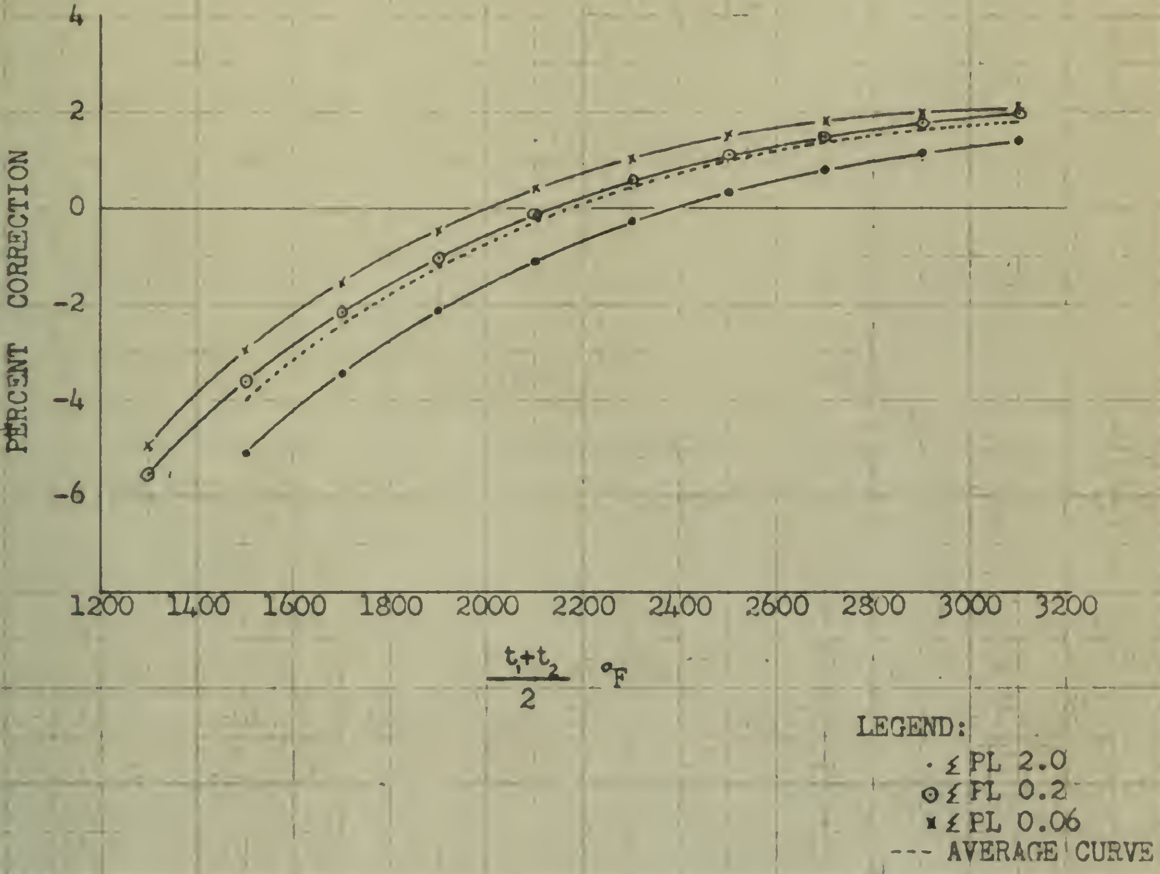
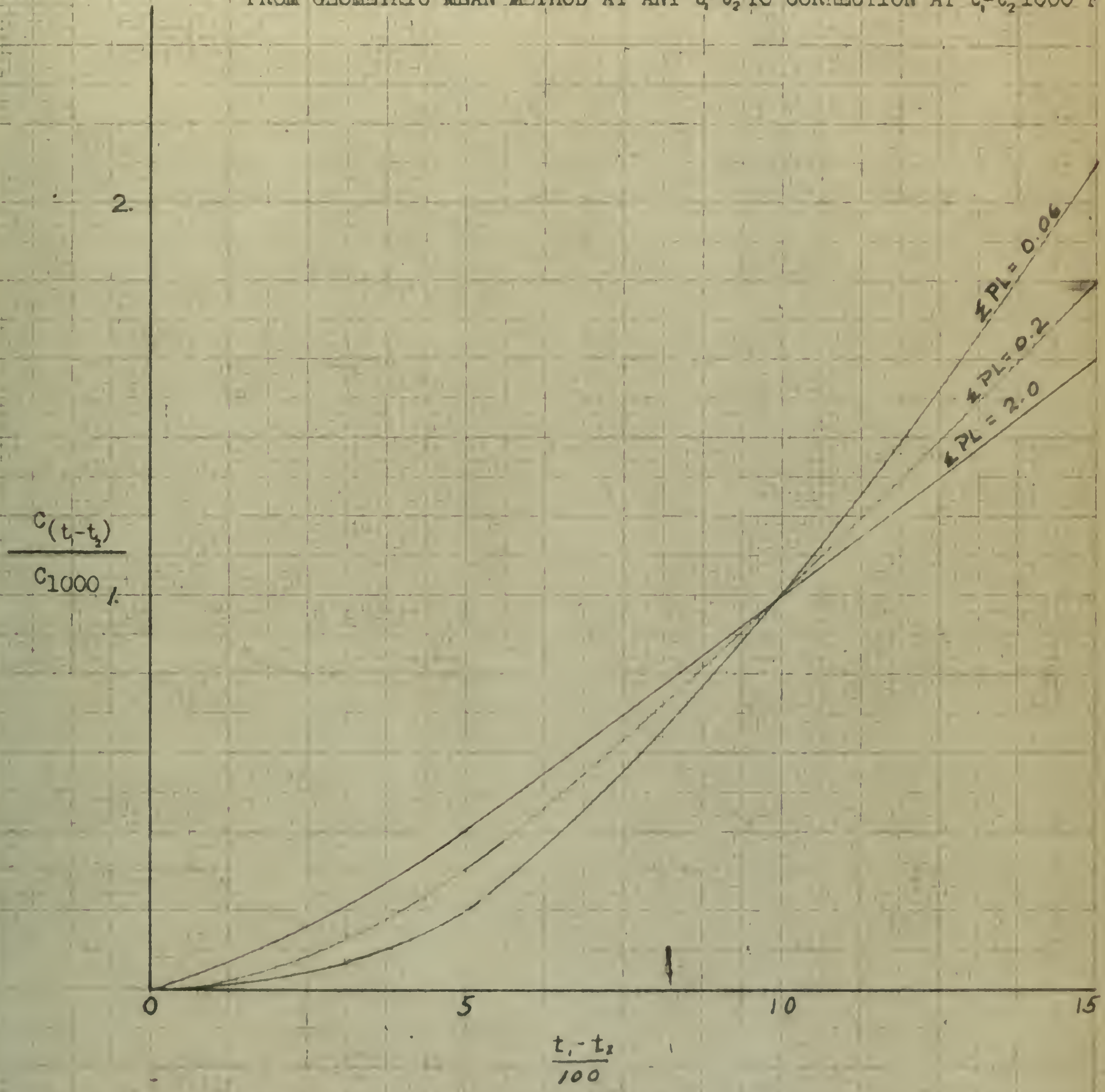


FIGURE XVII
RATIO OF CORRECTION FORAVERAGE VALUE OF q/A
FROM GEOMETRIC MEAN METHOD AT ANY t_1-t_2 TO CORRECTION AT t_1-t_2=1000 F



KEY TO FIGURES XVIII, XIX, XX, XXI

SURFACE TEMPERATURE	100°F			1500°F			.8t. - 400°F		
εPL	2.0	0.2	.06	2.0	0.2	.06	2.0	0.2	.06
$\frac{P_w}{P_c + P_w} = 0$									
$\frac{P_w}{P_c + P_w} = 0.5$									
$\frac{P_w}{P_c + P_w} = 1.0$									

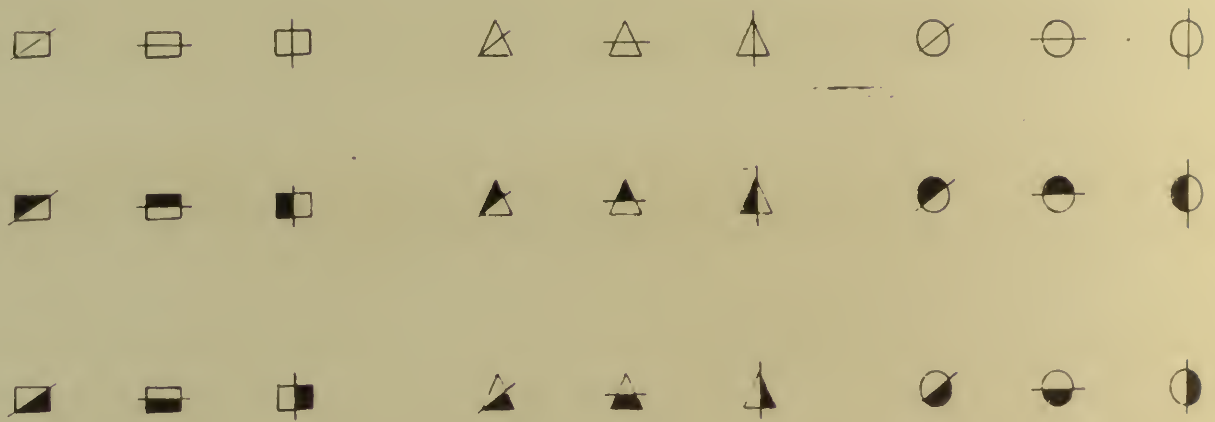
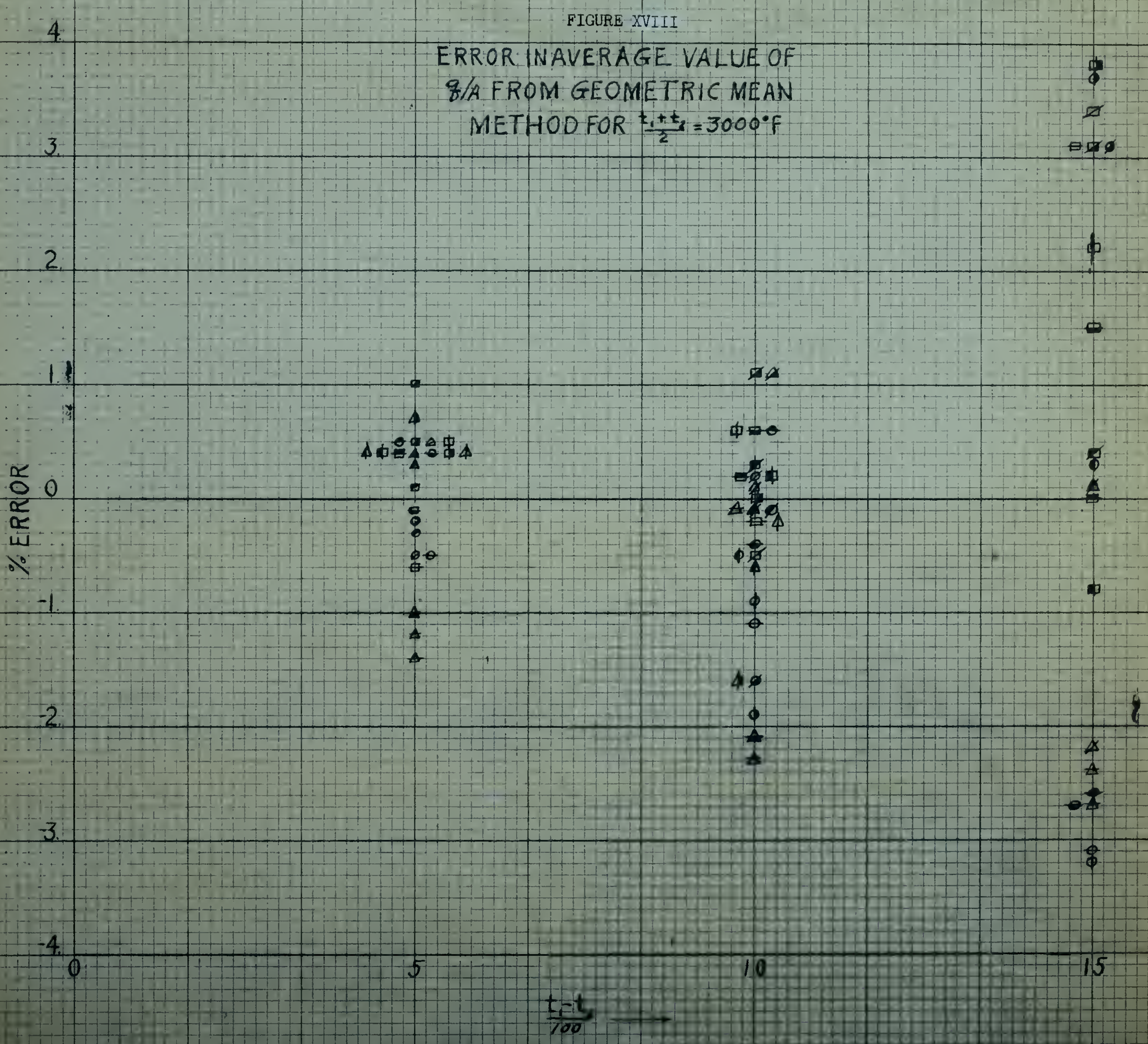


FIGURE XVIII



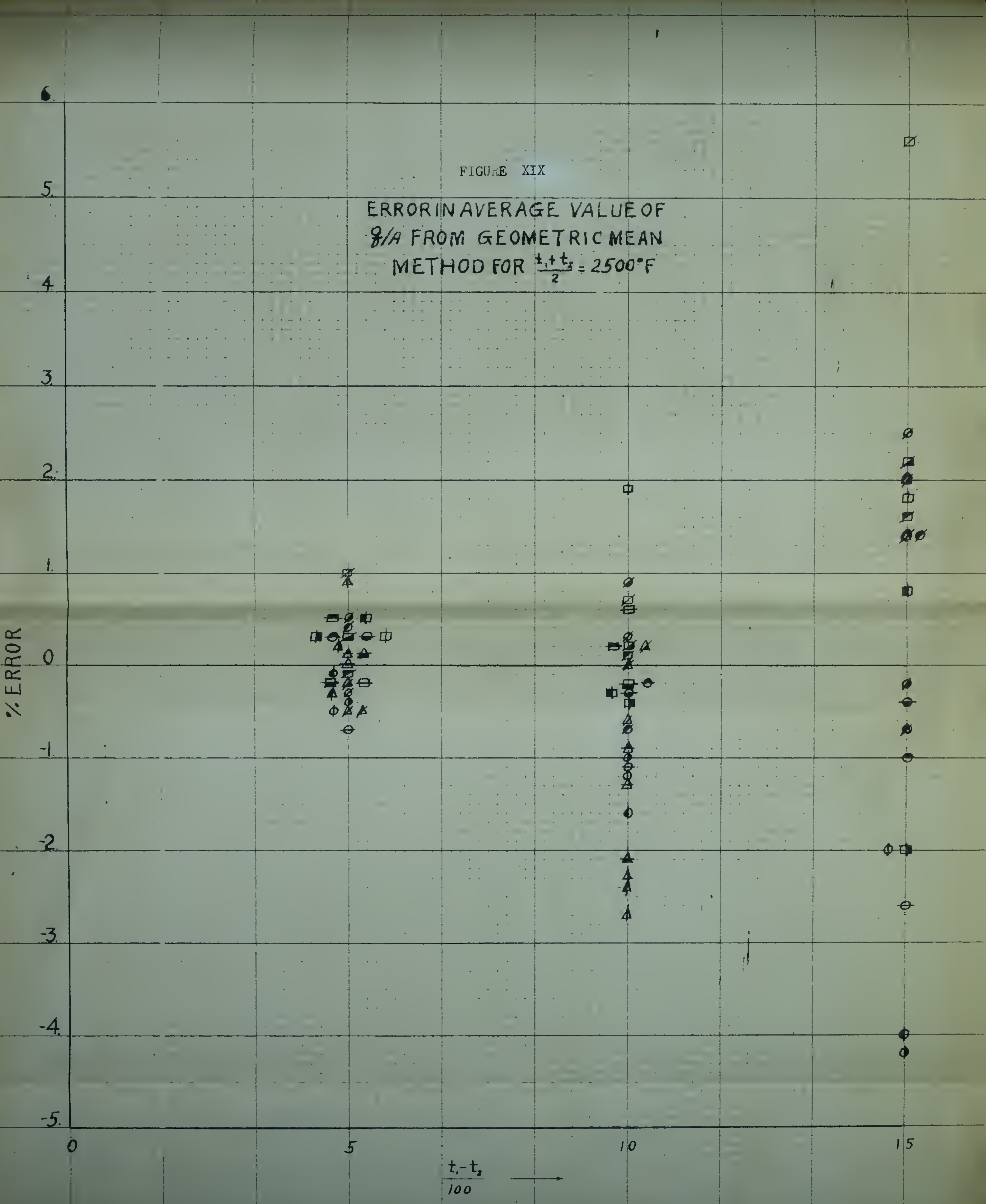


FIGURE XIX

ERROR IN AVERAGE VALUE OF q/A FROM GEOMETRIC MEAN METHOD FOR $\frac{t_1+t_2}{2} = 2500°F$

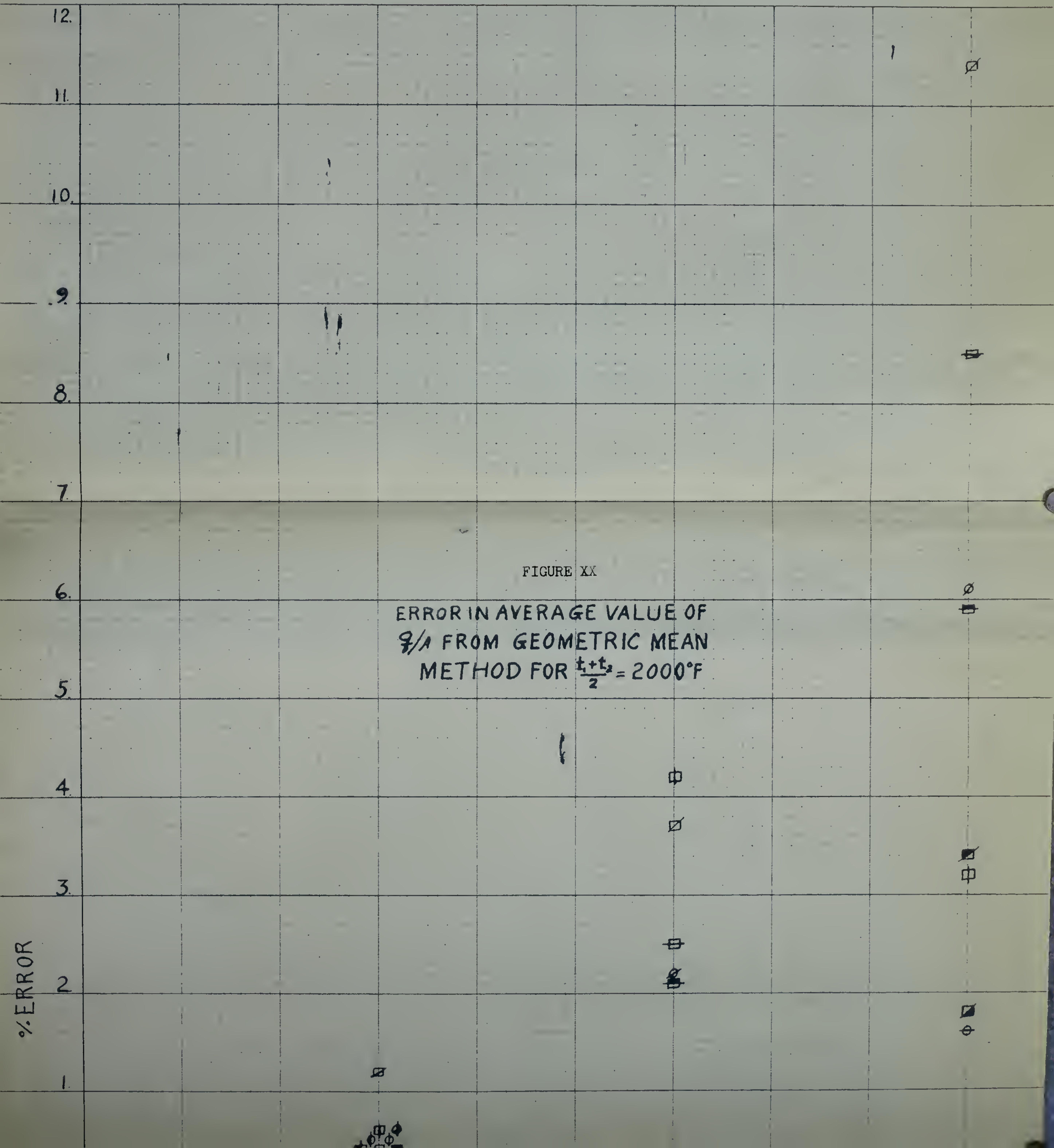
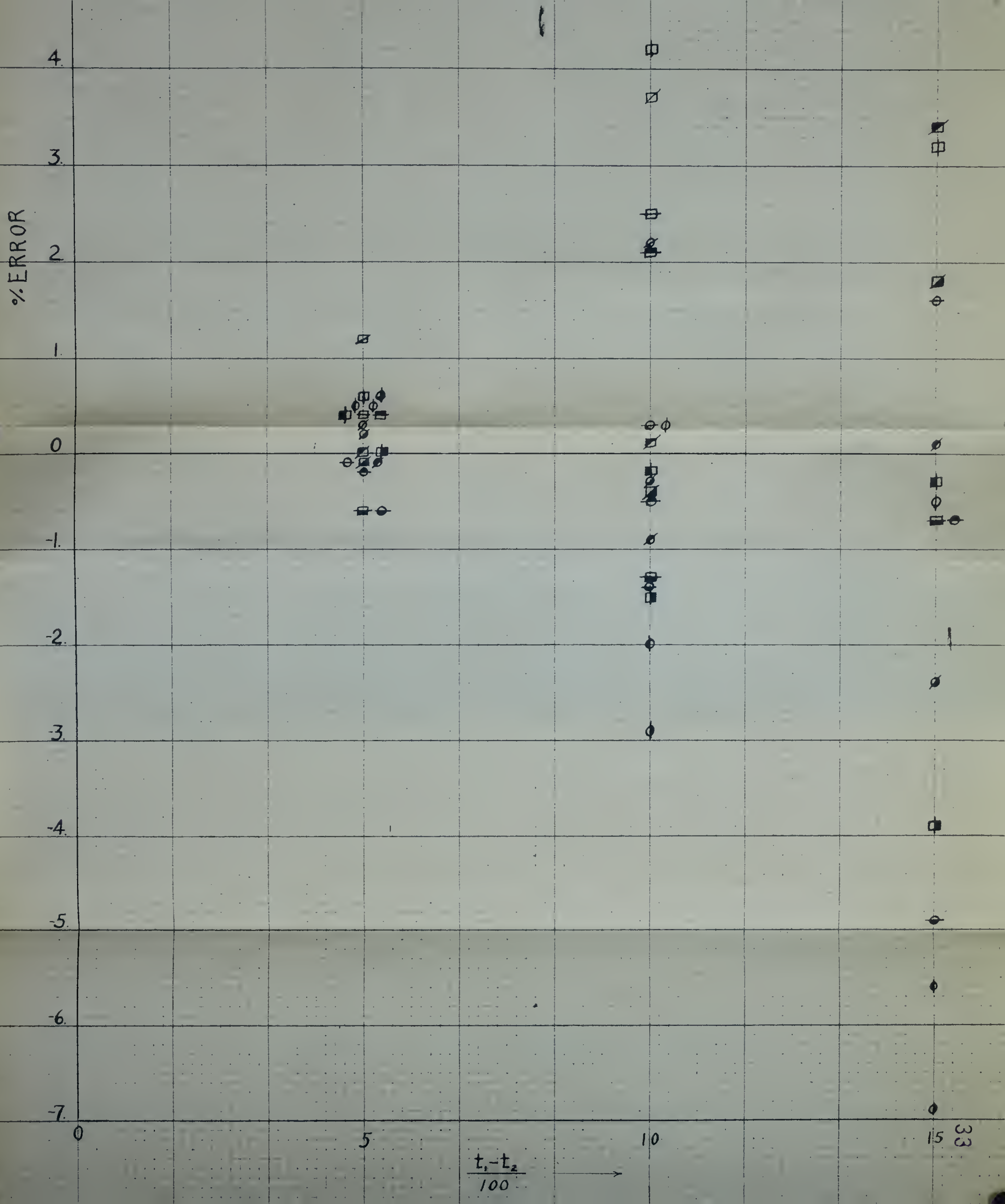


FIGURE XX

ERROR IN AVERAGE VALUE OF q/A FROM GEOMETRIC MEAN METHOD FOR $\frac{t_1+t_2}{2} = 2000°F$


% ERROR
4.
3.
2.
1.
0
-1.
-2.
-3.
-4.
-5.
-6.
-7.
0
5
10
15
$\frac{t_1-t_2}{100}$ →

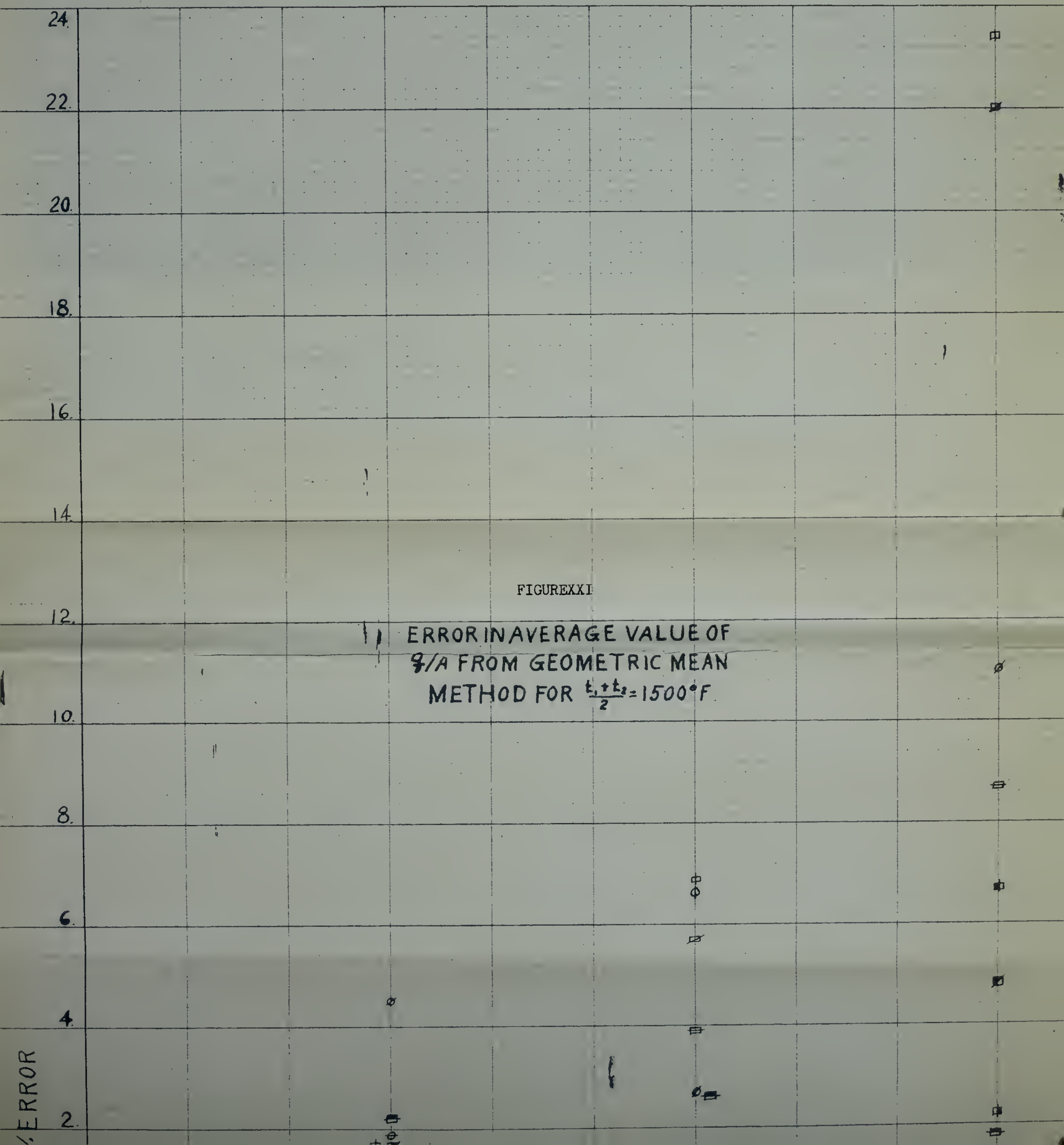
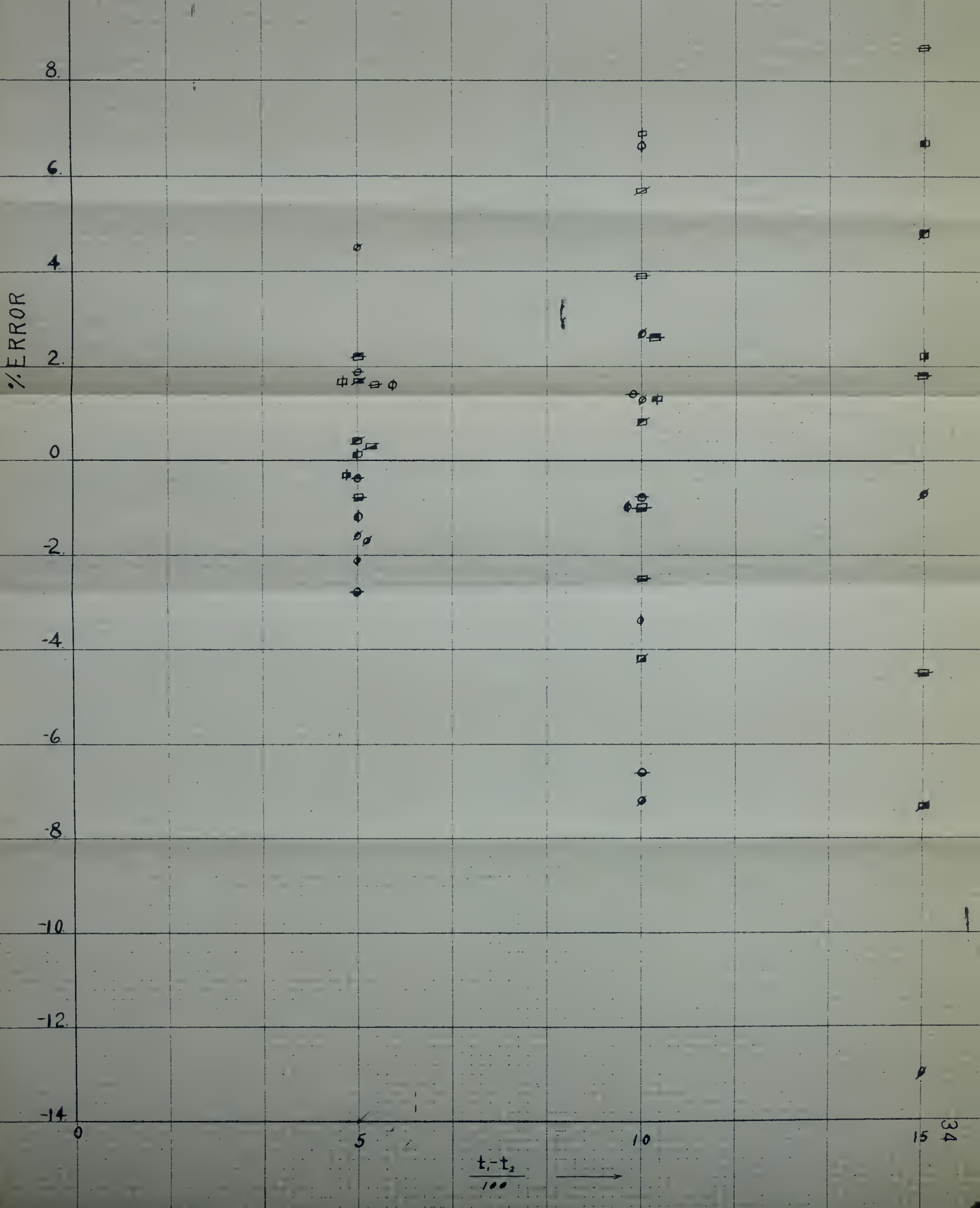


FIGURE XXI

ERROR IN AVERAGE VALUE OF q/A FROM GEOMETRIC MEAN METHOD FOR $\frac{t_1+t_2}{2} = 1500$°F


% ERROR
8.
6.
4.
2.
0
-2.
-4.
-6.
-8.
-10.
-12.
-14.
0
5
10
15
$\frac{t_1 - t_2}{100}$

DISCUSSION OF RESULTS

The curves of figures III, IV, and V which compare the correction to be applied to the average heat transfer rate obtained from the three methods investigated show that the geometric mean method permits elimination of surface temperature as a variable with introduction of the least error in the heat transfer rate.

At this point it is possible to observe the error in the calculation. Faired curves drawn to represent one surface temperature condition do not pass through all the computed points. The deviation of a point from its associated curve is a measure then of the error of calculation. The percent error is very nearly equal to this difference in percent correction, and thus may be observed by reference to figures III through XIV. The maximum calculation error is % and the average value is %.

Having determined the trend of the true correction curves required for use of the geometric mean method at several surface temperatures, an average correction curve is drawn to permit the elimination of surface temperature as a variable. The average curve for $t_1 - t_2 = 1000$ is seen to be a good representation of the several curves for different surface temperature conditions. An exception is necessary for the case where the difference between gas and surface temperatures is less than 500°F. The average curves for $t_1 - t_2 = 500$ and for $t_1 - t_2 = 1500$ are not in all cases the best representation of the several curves for different surface temperatures. They are derived from the corresponding average curve for $t_1 - t_2 =$ 1000 and from the average ordinate ratios given in figure XVII.

In order to further simplify the correction curves for engineering use all the correction curves have been finally averaged into two recommended curves, one for $t_1 - t_2 = 1000$ and one to convert this curve for any value of $t_1 - t_2$, (figures I-A and I-B). The recommended values have been plotted back with the true corrections on figures VI through XIV. They are seen to represent the true corrections well at high gas temperatures but poorly when low gas temperatures are combined with high values of $t_1 - t_2$.

The amount of error that is incurred from use of the geometric mean method and the recommended correction curves plotted as bands of possible error in figure II is seen to be reasonably symmetrical about zero for all values of $t_1 - t_2$. This degree of symmetry substantiates the recommended curves as being a good mean of corrections for any condition that may be imposed.

The conditions imposed of unit tube surface emissivity and 100% heat receiving surface should not affect the character of the recommended methods of calculating the average heat transfer rate. Likewise neglecting the correction for partial pressure of water vapor should have no effect on the recommendations.

CONCLUSIONS AND RECOMMENDATIONS

It is practical to calculate the average radiant heat transfer rate from gaseous carbon dioxide and water vapor flowing through a long thin tube by use of the geometric mean method and the recommended correction curves, figures I-A and I-B. Such a calculation will involve a possible error in the order of 5% or less for the most frequent range of industrial radiant heat transfer. The bands of possible error should be consulted by the designer, however, so that he may know what degree of accuracy he can expect from use of the recommended method. A limitation is placed on the method that it shall not be employed when the difference between gas and surface temperatures is less than 500°F.

Whether or not the conditions for no axial heat exchange and for complete normal mixing are sufficiently satisfied in a particular case is a subject which must rest in the hands of the designer. The condition that the effect of convection heat transfer is considered negligible will hold with varying degrees of accuracy depending on the situation. Convection heat transfer independently of the radiant heat transfer, and finally superimposing the convection upon the radiant heat transfer. When convection is an appreciable fraction of the total heat transfer a more rigorous solution will be necessary and the use of the geometrical mean method of predicting the radiant heat transfer cannot be recommended.

APPENDIX

APPENDIX

DETAILED PROCEDURE AND SAMPLE CALCULATIONS

The detailed procedure of calculation is given below together with sample calculations for each step.

Case: $P_T = 1$ atmosphere

$\Sigma PL = 2.0$ foot atmospheres

$\frac{P_w}{P_c + P_w} = 0.5$

$t_s = 100\ ^{\circ}F$, constant

$\epsilon_s = 1$

$t_1 - t_2 = 1000^{\circ}F$

1. Calculate the heat transfer at a point.

$t_F = 2500^{\circ}F$	$T_F = 2960^{\circ}F$
$t_s = 100^{\circ}F$	$T_s = 560^{\circ}F$
$P_wL = P_cL$	$P_wL + P_cL = 2.0$

$P_wL = P_cL = 1.0$ ft. atm.

Values of emissivity are found from the charts for water vapor and carbon dioxide (3) neglecting the correction for P_w.

$$\epsilon_c, T_F = .116$$

$$\epsilon_w, T_F, P_w = 0 \quad = .145$$

$$\Delta\epsilon_G, T_F = -.048$$

$$\epsilon_G = .213$$

$$\frac{T_s}{T_F} = \frac{560}{2960} = .189 \qquad P_cL\,\frac{T_s}{T_F} = 1 \times .189 = .189$$

$$\epsilon_c,\ T_s,\ P_cL \times \frac{T_s}{T_F} = .085$$

$$\frac{T_F}{T_s} = \frac{2960}{560} = 5.28 \quad \left(\frac{T_F}{T_s}\right)^{.65} = (5.28)^{.65} = 2.94$$

$$\alpha_c = \epsilon_c\ ,\ T_s,\ P_cL \times \frac{T_s}{T_F}\left(\frac{T_F}{T_s}\right)^{.65} = \quad .085 \times 2.94 = .250$$

$$\alpha_w = \epsilon_w,\ T_s,\ P_w = 0 \qquad = .328$$

$$\Delta\epsilon_{G,T_s} \qquad = -.018$$

$$\alpha_{G,\ s} \qquad = .560$$

$$\left(\frac{T_F}{100}\right)^4 = 29.60^4 = 768 \times 10^3$$

$$\epsilon_G\left(\frac{T_F}{100}\right)^4 = .213 \times 768 \times 10^3 = 164 \times 10^3$$

$$\left(\frac{T_s}{100}\right)^4 = 5.60^{\ 4} = .983 \times 10^3$$

$$\alpha_{G,\ s}\left(\frac{T_s}{100}\right)^4 = .560 \times .983 \times 10^3 = .550 \times 10^3$$

$$q/A\sigma = \epsilon_G\,\frac{T_F}{100}^{\ 4} - \alpha_{G,\ s}\,\frac{T_s}{100}^{\ 4}$$

$$= (164 - .550) \times 10^3 = 163 \times 10^3$$

$$\frac{1}{q/A\sigma} = .00610 \times 10^{-3}$$

Similarly other points are calculated and plotted against gas temperature as shown in figure XXII.

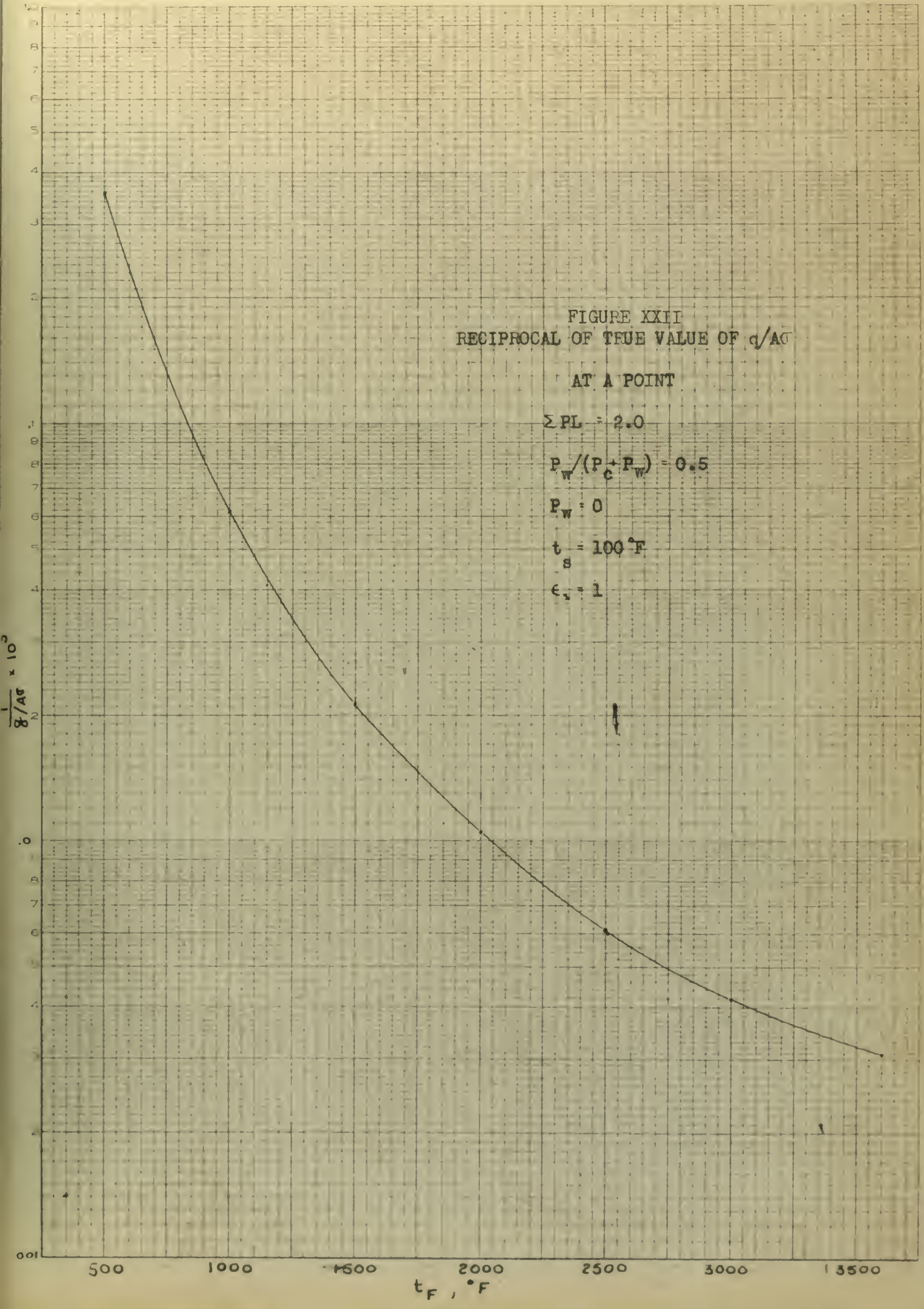


FIGURE XXII
RECIPROCAL OF TRUE VALUE OF q/Aσ
AT A POINT

$\Sigma PL = 2.0$

$P_W/(P_C + P_W) = 0.5$

$P_W = 0$

$t_s = 100°F$

$\epsilon_s = 1$

2. Calculate the true average heat transfer over a length of tube where the gas temperature varies a given amount.

The average value of $q/A\sigma$ is given by

$$(q/A\sigma)_{ave} = \frac{T_1 - T_2}{\int_{T_2}^{T_1} \frac{dT}{q/A\sigma}}$$

The integral may be evaluated by plotting $\frac{1}{q/A\sigma}$ versus T_F. Ordinates are then picked off at 100 degree intervals and summed.

Case: $t_1 - t_2 = 1000\ ^\circ F$ $\quad \frac{t_1 + t_2}{2} = 2100\ ^\circ F$

t_F	$\frac{1}{q/A\sigma} \times 10^6$
1650	16.5
1750	14.0
1850	12.3
1950	10.8
2050	9.6
2150	8.6
2250	7.7
2350	7.0
2450	6.4
2550	5.9
	98.8

$$(q/A\sigma)_{ave} = \frac{(t_1 - t_2)/100}{\Sigma \frac{1}{q/A\sigma}} = \frac{10}{98.8 \times 10^{-6}} = 101 \times 10^3$$

Other points are calculated in the same manner. They are plotted against the arithmetic mean gas temperature in figure XXIII as a check for accuracy.

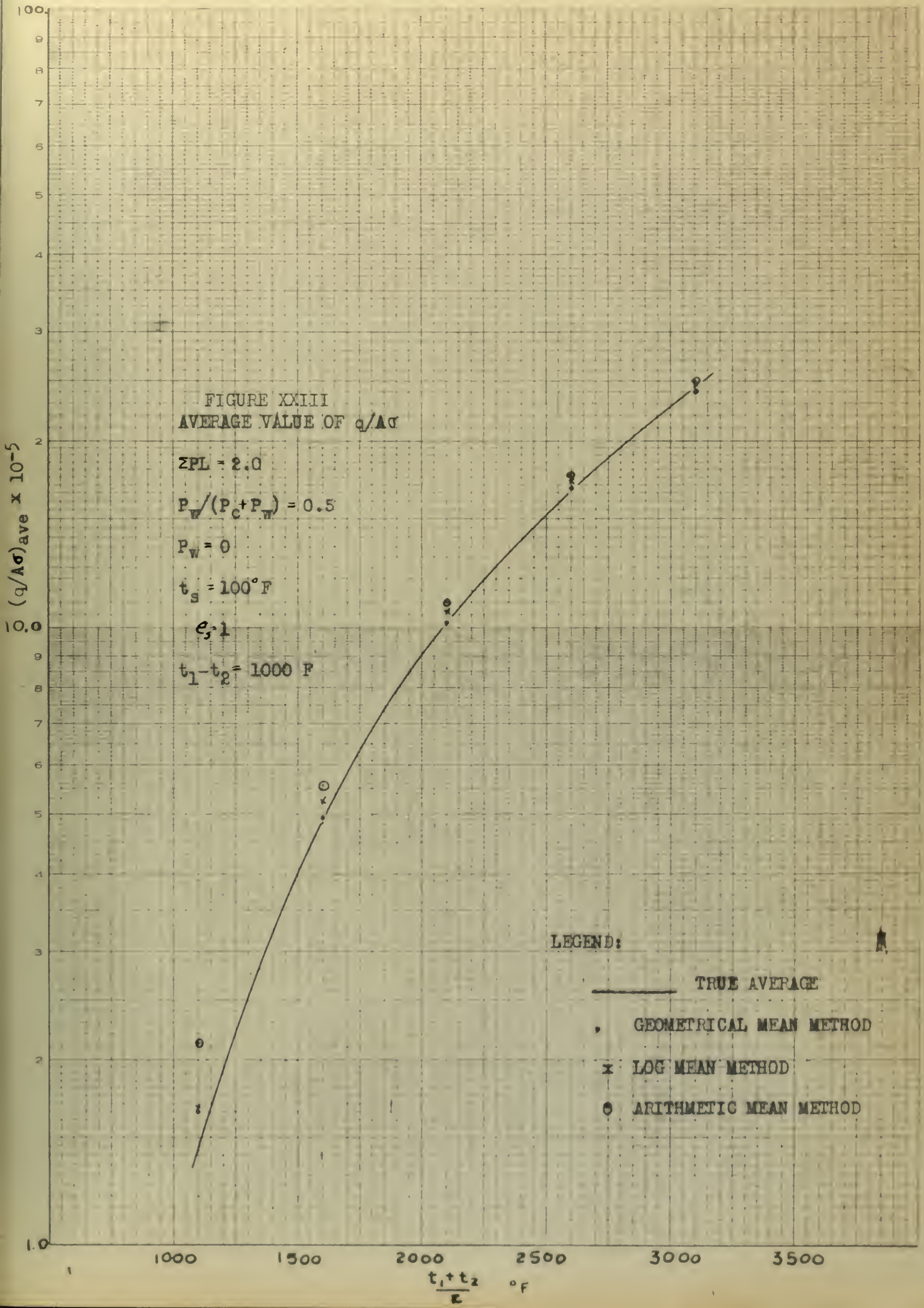


FIGURE XXIII
AVERAGE VALUE OF q/Aσ

$\Sigma PL = 2.0$

$P_W/(P_C+P_W) = 0.5$

$P_W = 0$

$t_s = 100°F$

$e_s = 1$

$t_1-t_2 = 1000$ F

3. Calculate the average heat transfer by the geometric mean method.

$$(q/A\sigma)_{ave} = \mathcal{F}_{ave}\left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]_{ave}$$

$\mathcal{F}_{ave}$ is calculated at $\frac{t_1 + t_2}{2}$

$\left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]_{ave}$ = geometric mean of inlet and outlet values.

Case: $t_1 - t_2 = 1000\ ^\circ F$ $\qquad \frac{t_1 + t_2}{2} = 2100\ ^\circ F$

$t_1 = 2600^\circ F$ $\qquad T_1 = 3060\ ^\circ F$ abs

$t_2 = 1600\ ^\circ F$ $\qquad T_2 = 2060\ ^\circ F$ abs

$t_{si} = t_{so} = 100\ ^\circ F$ $\qquad T_{si} = T_{so} = 560\ ^\circ F$ abs

From plot of step 1 pick off

$$1/(q\ /A\sigma) = 9.10 \times 10^{-6} \text{ at } \frac{t_1 + t_2}{2} = 2\,100\ ^\circ F$$

$$q/A\sigma = 110 \times 10^3$$

$$T_{F,\ ave} = \frac{T_1 + T_2}{2} = 2560\ ^\circ F \text{ abs}$$

$$\left[\frac{T_{F,\ ave}}{100}\right]^4 = 25.60^4 = 429 \times 10^3$$

$$T_{s,\ ave} = 560\ ^\circ F \text{ abs}$$

$$\left(\frac{T_{s,\ ave}}{100}\right)^4 = 5.60^4 = .983 \times 10^3$$

$$\mathcal{F}_{ave} = \frac{q/A\sigma}{\left(\frac{T_{F,\ ave}}{100}\right)^4 - \left(\frac{T_{s,\ ave}}{100}\right)^4} = \frac{110 \times 10^3}{428 \times 10^3} = .257$$

$$\left(\frac{T_1}{100}\right)^4 = 30.6^{\,4} = 875 \times 10^3 \qquad \left(\frac{T_2}{100}\right)^4 = 20.6^{\,4} = 181 \times 10^3$$

$$\left(\frac{T_{so}}{100}\right)^4 = 1 \times 10^3 \qquad \left(\frac{T_{si}}{100}\right)^4 = 1 \times 10^3$$

$$\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_{so}}{100}\right)^4 = 874 \times 10^3 \qquad \left(\frac{T_2}{100}\right)^4 - \left(\frac{T_{si}}{100}\right)^4 = 180 \times 10^3$$

$$\left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]_{ave} = \sqrt{\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_{so}}{100}\right)^4\right]\left[\left(\frac{T_2}{100}\right)^4 - \left(\frac{T_{si}}{100}\right)^4\right]}$$

$$= \sqrt{(874 \times 10^3)\ (180 \times 10^3)}$$

$$= 397 \times 10^3$$

$$(q/A\sigma)_{ave} = \mathcal{F}_{ave}\left[\left(\frac{T_F}{100}\right)^4 - \left(\frac{T_s}{100}\right)^4\right]_{ave}$$

$$= .257 \times 3\ 97 \times 10^3 = 102 \times 10^3$$

Similarly othe r values of $(q/A\sigma)_{ave}$ are calculated by this method and are plotted with the true v alue of $(q/A\sigma)_{ave}$ as a check for accuracy, figure XXIII.

4 . C alculate the average heat transfer by the arithmetic mean method.

$$(q/A\sigma)_{ave} = \mathcal{F}_{ave}\left[\left(\frac{T_{F,ave}}{100}\right)^4 - \left(\frac{T_{s,ave}}{100}\right)^4\right]$$

$\mathcal{F}_{ave}$ is calculated at $t_{F,ave}$ and $t_{s,ave}$

$$t_{F,ave} = \frac{t_1 + t_2}{2} \qquad t_{s,ave} = \frac{t_{si} + t_{so}}{2}$$

Case: $t_1 - t_2 = 1000\,^\circ F$ $\qquad \frac{t_1 + t_2}{2} = 2100\ ^\circ F$

From curve of step 1 at 2100 °F pick off $\frac{1}{q/A\sigma} = .00910 \times 10^{-3}$

$$(q/A\sigma)_{ave} = (q/A\sigma) = 110 \times 10^3$$

Similarly other values of $(q/A\sigma)_{ave}$ are calculated by this method

and are plotted with the true average as a check for accuracy.

5. Calculate the average heat transfer by the log mean method for the case of step 2.

$$(q/A\sigma)_{ave} = \mathcal{F}_{ave} \left[\left(\frac{T_{F,AVE}}{100}\right)^4 - \left(\frac{T_{S,AVE}}{100}\right)^4 \right]$$

$\mathcal{F}_{ave}$ is calculated at $t_{F,ave}$ and $t_{s,ave}$

$$t_{s,ave} = \frac{t_{si} + t_{so}}{2} \qquad t_{F,ave} = t_{s,ave} + \text{log mean } \Delta t$$

$$t_1 - t_2 = 1000^\circ F \qquad \frac{t_1 + t_2}{2} = 2100^\circ F$$

$$t_1 = 2600^\circ F \qquad t_2 = 1600^\circ F$$

$$t_{so} = 100^\circ F \qquad t_{si} = 100^\circ F$$

$$\Delta t_1 = 2500^\circ F \qquad \Delta t_2 = 1500^\circ F$$

$$\Delta t_1 - \Delta t_2 = 1000^\circ F$$

$$\frac{\Delta t_1}{\Delta t_1} = \frac{2500}{1500} = 1.67$$

$$\ln \frac{\Delta t_1}{\Delta t_2} = .515$$

$$\Delta t_m = \frac{\Delta t_1 - \Delta t_2}{\ln \frac{\Delta t_1}{\Delta t_2}} = \frac{1000}{.515} = 1950$$

$$t_{F,ave} = t_{s,ave} + \Delta t_m = 100 + 1950 = 2050^\circ F$$

From curve of step 1 pick off

$$\frac{1}{(q/A\sigma)} = .00960 \times 10^{-3} \quad \text{at } t_F = 2050^\circ F$$

$$(q/A\sigma)_{ave} = 1/.00960 \times 10^{-3} = 104 \times 10^{-3}$$

Similarly other values of $(q/A\sigma)_{ave}$ are calculated by this method and plotted with the true average as a check for accuracy.

6. Calculate the percent correction for the average value of $(q/A\sigma)$ from the three methods.

$$(1+\%C/100)\ (q/A\sigma)_{ave,\ method} = (q/A\sigma)_{ave,\ true}$$

$$\%C/100 = \frac{(q/A)_{ave.\ true}}{(q/A)_{ave,\ method}} - 1$$

Case: True $(q/A)_{ave} = 101 \times 10^3 \cdot \sigma$ (step 2)

Geometric mean method $(q/A)_{ave} = 102 \times 10^3 \cdot \sigma$ (step 5)

$\%C = 100\ (101/102 - 1) = -1.0\%$

Arithmetic mean method $(q/A\sigma)_{ave} = 110 \times 10^3$ (step 4)

$\%c = 100\ (101/110 - 1) = -8.2\ \%$

Log mean method $(q/A\sigma)_{ave} = 104 \times 10^3$ (step 5)

$\%C = 100\ (101/104 - 1) = -2.9\ \%$

7. Evaluate the recommended ratio of correction at any value of (t_1-t_2) to correction at $(t_1-t_2) = 1000°F$ for $(q/A)_{ave}$ from the geometric mean method.

Plot against $(t_1+t_2)/2$ all values of percent correction at one and one $P_w/(P_c+P_w)$ for the desired value of (t_1-t_2) and for $(t_1-t_2) = 1000°F$, figure VII

Fair a curve through points representing each surface temperature condition. Fair the best average curve through these curves for each (t_1-t_2) forcing the average curves to go through zero at the same $(t_1+t_2)/2$. (Shown only for $(t_1-t_2) = 1000°F$ (figure VII.) Calculate ratios of ordinates at the same $(t_1+t_2)/2$ and take the arithmetic average of the ratios.

Similarly find this average ratio for other values of $P_w/(P_c+P_w)$ at the same ΣPL.

Find arithmetic average of this correction ratio for all values of $P_w/(P_c+P_w)$ at each ΣPL.

Again average these arithmetically for the recommended ratio.

Case: $\Sigma PL = 2.0$ $P_w/(P_c+P_w) = 0.5$

$(t_1+t_2)/2$	$\%C_{1000}$	$\%C_{1500}$	$\frac{\%C_{1500}}{\%C_{1000}}$
3000	1.5	1.8	1.2
2800	1.4	2.2	1.6
2600	1.0	1.7	1.7
2400	.4	.7	1.8
2300	0	0	-
2200	-0.5	-0.7	1.4
2000	-1.5	-2.5	1.7
1800	-2.7	-4.3	1.6
			11.0

$$\frac{\%C_{1500}}{\%C_{1000}}\text{ ave} = \frac{11.0}{7} = 1.6$$

$$P_w/(P_c\ P_w) = 0 \quad \frac{\%C_{1500}}{\%C_{1000}}\text{ ave} = 1.7$$

$$P_w/(P_c\ P_w) = 1.0 \quad \frac{\%C_{1500}}{\%C_{1000}}\text{ ave} = 1.5$$

$$\Sigma PL = 2.0 \quad \frac{\%C_{1500}}{\%C_{1000}} = 1.0$$

$$\Sigma PL = 2.0 \qquad \frac{\%C_{1500}}{\%C_{1000}}_{\text{ave}} = 1.6 \qquad \text{(see figure XVII)}$$

$$\Sigma PL = 0.20 \qquad \frac{\%C_{1500}}{\%C_{1000}}_{\text{ave}} = 1.8$$

$$\Sigma PL = 0.06 \qquad \frac{\%C_{1500}}{\%C_{1000}}_{\text{ave}} = 2.1$$

$$\text{Recommended} \qquad \frac{\%C_{1500}}{\%C_{1000}}_{\text{ave}} = 1.8 \qquad \text{(see figure I-B)}$$

8. Evaluate recommended correction curve for $(t_1-t_2) = 1000\ ^{\circ}F$.

Plot average correction curves for one ΣPL and all values of $P_W/(P_C+P_W)$ together and average their ordinates, figure IV.

Case: $\Sigma PL = 2.0 \quad (t_1+t_2)/2 = 2100\ ^{\circ}F$

$P_W/(P_C+P_W)$	0	0.5	1	Ave
%C	-2.5	-0.4	-0.5	-1.1

Plot the resulting average correction curves for all ΣPL's together and average their ordinates.

Case: $(t_1+t_2)/2 = 2100^{\circ}F$

ΣPL	2.0	0.20	0.06	Ave
%C	-1.1	-0.1	0.5	-0.2

This final average curve is plotted as the recommended correction curve, figure I-A.

9. Calculate the percent error in $(q/A)_{\text{ave}}$ from the geometric mean method and the recommended correction.

$$\frac{\%E}{100} = \frac{(q/A)_{\text{ave, recommended}} - (q/A)_{\text{ave, true}}}{(q/A)_{\text{ave, true}}}$$

$$= \frac{(q/A)_{\text{ave, method}} \left[\left(1 + \frac{\%C_{\text{rec}}}{100}\right) - \left(1 + \frac{\%C_{\text{true}}}{100}\right) \right]}{(q/A)_{\text{ave, method}} \quad 1 \quad \frac{\%C_{\text{true}}}{100}}$$

Case: $\%C_{\text{true}} = -1.0\ \%$ (step 6)

$\%C_{\text{recommended}} = -0.2\ \%$ (step 8)

$$\%E = 100\ \frac{(-0.2 \quad 1.0)/100}{1 \quad (-1.0/100)} = -0.8/0.99 = -0.8\ \%$$

(see figure XVIII)

Plot the error for all combinations studied at several representative temperatures and present the bands of possible error, figure II.

APPENDIX

DATA AND CALCULATIONS

All calculations of gas emissivity and absorptivity are based on the recommendations of Hottel and Egbert (2) who present data on the emissivities of water vapor and carbon dioxide together with allowances for their mutual interference when mixed.

The calculations supporting the results are contained in two books, which are in the custody of Professor H. C. Hottel at Massachusetts Institute of Technology.

APPENDIX

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